Using a Wiki to Enhance Cooperative Learning in a Real Analysis Course

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Introduction

For decades, educators have become increasingly convinced of the importance of cooperative learning in the classroom, and have sought out new strategies to accomplish this goal. It is widely believed that cooperative learning increases the ability of students to work together, keeps them more engaged in the process, and better reinforces the material being learned.

In this paper we investigate the efficacy of a wiki as a tool for collaborative learning and enhancing students ability to communicate in an undergraduate mathematics course. A *wiki* is a website which is constructed primarily for the purposes of online authoring and collaboration [8]. The prototypical example is Wikipedia, the online encyclopedia constructed with the help of tens of thousands of users. But wikis also exist on much smaller scales. There are thousands of wiki sites on the web, allowing groups of individuals to collaborate around shared interests such as an employer, video games, or mathematics. Sometimes wikis are created for use by a single person. (This paper was written on the author's *personal wiki*.) There are several features which attract users to wikis:

- *Online authoring*: a single document may be edited anywhere the Internet is accessible
- *Collaborative authoring*: multiple users may easily edit the same document, and user-access may be controlled
- *Simplified markup language*: documents may be edited without needing to learn a "complicated" markup language like HTML
- *Mathematical typesetting*: many wikis have the ability to typeset mathematics using LaTeX
- *Linking structure*: wikis are built as collections of pages with an abundance of links between them; frequently the linking is done automatically

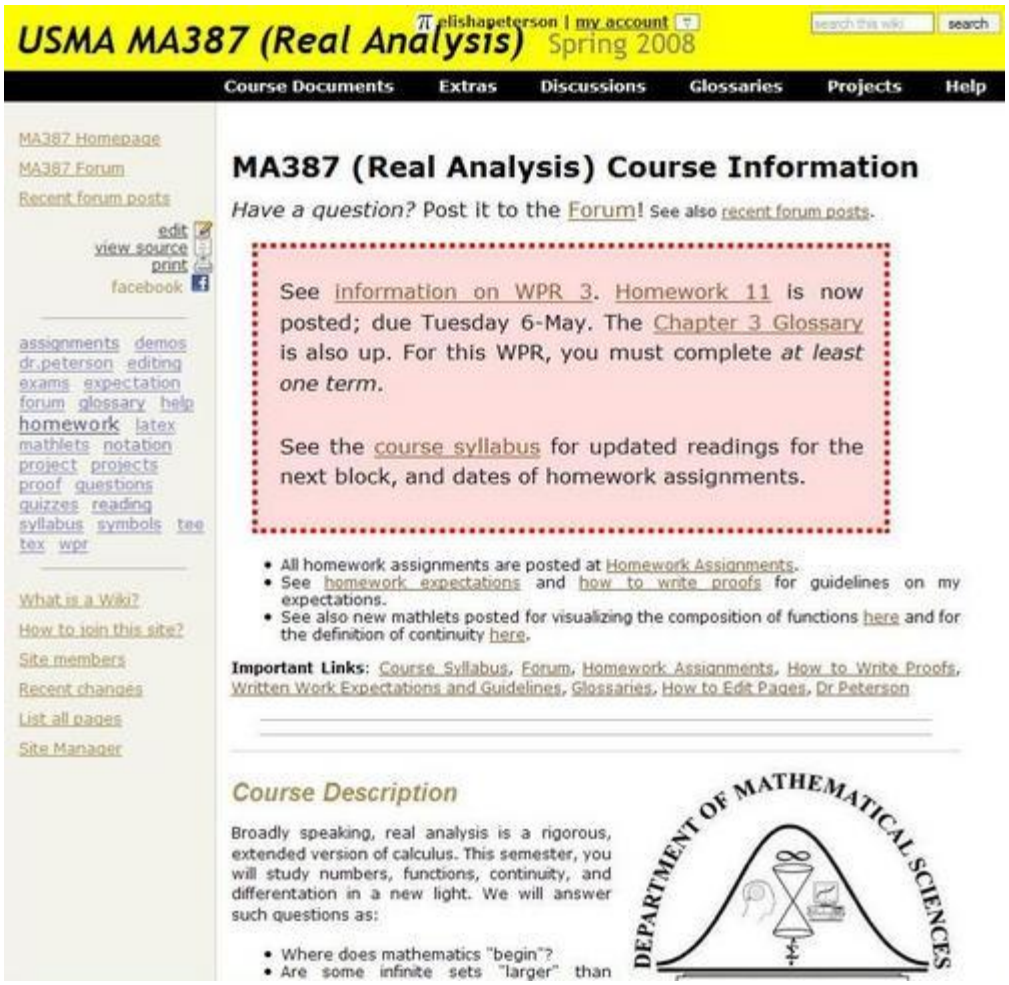


Figure 1. Screen shot of course wiki home page.

For all of these reasons, it is natural to assume that a wiki would make a good classroom tool as well. In the classroom, cooperative learning and collaborative authoring are frequently course objectives. In addition, the development of knowledge in the world today is increasingly distributed. Wiki-like models have been used not only for collaboratively constructing knowledge, but also for collaborative problem-solving [7].

To evaluate a wiki as a classroom tool, a wiki website was developed and maintained at http://usma387.wikidot.com for Real Analysis, an undergraduate mathematics major course. Wikidot is a free *wiki farm* allowing users to create their own access-controlled wiki sites. In the case of this course, students were given editing access to portions of the website and restricted from others. The wiki was used in four ways:

- Course materials were posted on the wiki, including the syllabus, homework assignments, solutions, and more.
- A forum was maintained at the course wiki.
- Student projects were posted to the course wiki.
- Students collaboratively developed a glossary of terms during each course block, which was provided to them during examinations.

The class using the wiki was very small, with only nine students with a wide spectrum of GPAs, rendering statistical analysis useless. To that end, students were surveyed regarding how the wiki was used in the class. These surveys, together with instructor insight, were used to assess the efficacy of the course wiki. We will examine each of the ways in which the wiki was used, and discuss any insight gained regarding the difference between a course wiki and a more traditional course management system such as Blackboard. The evidence that a course wiki enhanced the students experience in this course is dramatic, and indicates that wikis could be a powerful platform for collaborative learning in the future.

The initial objectives for the course wiki were the following:

- Encourage collaboration among the students.
- Develop students ability to communicate mathematics to others, and give them the tools to do so.
- Give students a "gentle" introduction to TeX and LaTeX.
- Expose students to collaborative technologies and the movement towards collaborative projects.
- Encourage students to explore more on their own by linking to other websites.

The primary initial concerns with the project were the lack of participation on the course wiki, a frequent problem with using course web features, as well as the learning curve required for students to be comfortable using the wiki. Another issue was instructor (and student) time management.

The Role of Real Analysis in the Undergraduate Math Major Curriculum

The real analysis course plays a fundamental role in the undergraduate math curriculum, and the effectiveness of the wiki cannot be adequately described without first describing the nature of the course. Real analysis is frequently the first or second major course taken by an undergraduate mathematics major, and plays a pivotal role in their development as mathematicians. In particular, it is usually their first or second exposure to *mathematical proof* and *rigor*. As the Department of Mathematical Sciences at

USMA presents few proofs in introductory Calculus courses, it is also frequently the students' first exposure to *epsilons and deltas*, that is, the rigorous treatment of limits. Initially, students find real analysis very challenging because it is unlike any course they have taken before. It is frequently their first exposure to *pure* mathematics. It is likely the most abstract subject they have seen, with an abundance of definitions and theorems rather than applications. It is the first opportunity they have to "do what pure mathematicians do".

Background on Learning Theory

In this section, we give a brief overview of cooperative learning and technology in the classroom. Wikis provide a unique opportunity to bring these two forces together.

Cooperative Learning

Cooperative learning, and small group work, is widely accepted as a good practice in teaching. Neil Davidson, Barbara Reynolds, and Elizabeth Rogers write regarding cooperative learning in mathematics:

> *"Learning can be a social activity, and mathematics is filled with exciting and challenging ideas for discussion. Students can learn by talking, listening, explaining, and thinking with others. Students are often able to explain ideas to one another using an informal language which is readily understood by their peers. In the very act of explaining or attempting to explain an idea, the student must reach for a deeper understanding of that idea. As students work together, they begin to recognize the need for more precise language to express their ideas. Once they have achieved deeper understanding and clarity, students are ready to adopt the more formal language of mathematical discourse that is used by their instructors and the authors of their textbooks."* [6]

For this reason, many universities have made the switch from large lectures to smaller courses in the introductory mathematics curriculum. Steven Krantz, a prominent mathematician and writer, states that such small class sizes "feel more empowered" about their learning, yet notes also that "no studies show significant improvement in learning, performance, or retention in such courses" [5].

Regardless, cooperative learning is not dependent on the size of the classroom, and does not require working in small groups, although much of the literature on cooperative learning in mathematics focuses on such.

Technology in the Classroom

Technology in the classroom is a highly active area of research in which few clear answers are known. As a general rule, the community of educators agrees that increased use of technology in the classroom is a good thing, enabling teachers to better connect with their students, and to better connect their students to the real world. But they frequently disagree on the kinds of technologies, or the levels of technologies, which should be used in the classroom [1, 4].

In mathematics, the Mathematical Association of America, the premier society of teachers of mathematics, hosts sessions on teaching with technologies at every national meeting. There are several international conferences devoted to technology and mathematics. One of the major areas of contention is the level to which *Computer Algebra Systems* should be used in the classroom. On the one hand, they offer the ability to solve much more difficult problems. On the other hand, they often have a steep learning curve for beginning college students, and they may prove as a distraction from the course requirements [1, 4].

Beyond computer algebra systems, there has also been discussion in recent years about using other kinds of technologies for teaching: PDA's, cell phones, MP3 players, podcasts, GPS devices, tablet PCs, and more. Work has already been published in many of these areas.

Emerging Web Technologies in the Classroom

Carole Barone makes the case that students today "think differently", being raised on gadgets and devices such as cell phones, instant messaging, and the Internet [2]. Social networking sites are known for the large chunks they take out of students' days. Instructors in all fields are increasingly looking for ways in which to leverage these new technologies for teaching. Some have gone so far as to hold class in the virtual world *Second Life*. Others have looked for ways for students to collaborate online.

Wiki Classroom Projects

There are also a smattering of online Wiki projects around the Internet. There are several "virtual classroom" wiki sites developed by instructors to provide their students with an online community. Some of these projects have proved to be the most active in the collection of wikis at *Wikidot*, the site hosting the wiki discussed in this paper. Some research has been done on the use of wikis in the classroom already, although the focus

of such projects is frequently more on building a collaborative knowledge base, similar to Wikipedia or a user-group wiki [3].

Project Description and Assessment

Course Materials and Forum

All course materials were maintained at the course wiki, including a syllabus which was updated over the course of the semester, homework assignments, and information about upcoming deadlines. One of the nicest features of the wiki was the ability to easily link to other sites, such as Wikipedia. This made it easy to provide students links to read about other applications of the subject, or the historical figures important in its development.

A forum for questions and answers by the students was maintained on the course wiki. Participation in this forum was completely optional, although categories were made for *Reading Questions*, *Homework Questions*, *Technology Questions*, and *Instructor Notes*. Students were invited to post questions to the forum, and the instructor posted homework solutions or hints there as well.

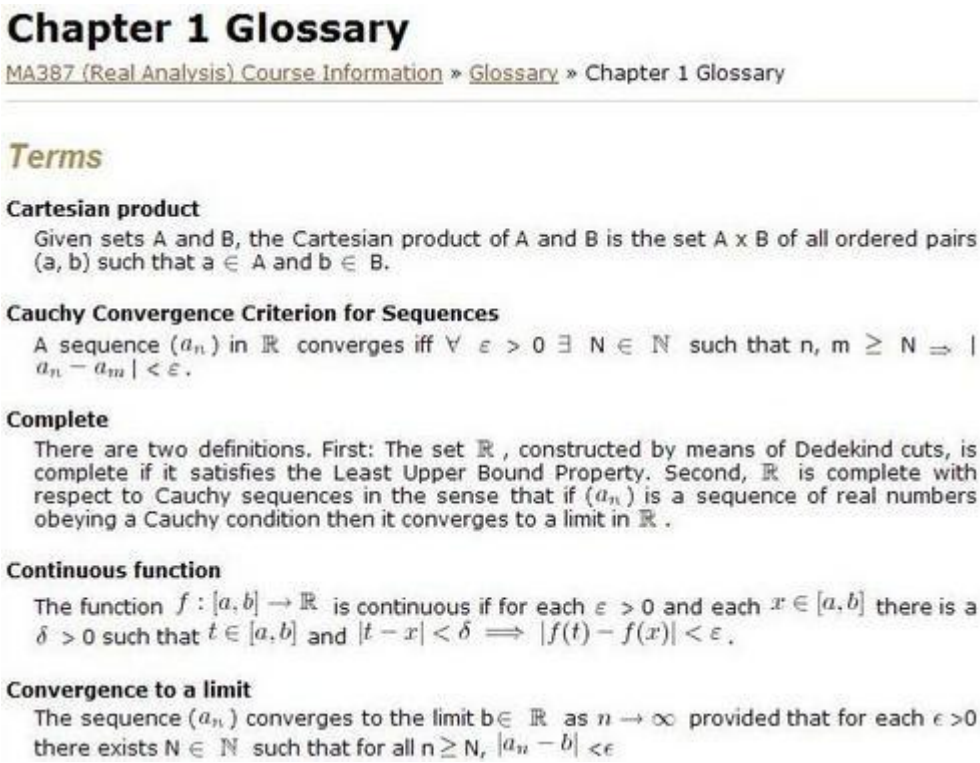
Assessment

Posting course materials to the web is nothing new, nor is maintaining a course forum. What is challenging, however, is getting students to use the course forum without a "stick or carrot". Although no requirement was made that students use the forum, many did post questions about the course forum, and the instructor used it quite often to communicate with the students. There is no easy solution to getting them more involved. Given that the class was so small, most of the students collaborated on their own apart from using the course forum. This is probably mostly due to the difficulty of the material. One student did report that he would have liked the forum to be used more, suggesting "more emphasis on working together using the wiki's forums" on a feedback survey after the first Block of the course (as a suggestion for improving the course). This indicates that perhaps, given the proper encouragement, students might have made more use of this feature.

Glossaries

The second, and most successful, way in which the course wiki was used was the creation of a glossary of terms during each block. Students were required to contribute three entries per block to the glossary, with a small amount of points assigned to this

task. The instructor contributed to the glossary by providing the "key terms" for the block and by organizing the terms. Students provided all the definitions. The students were highly motivated to contribute to the glossary as the final product of each block was made available to them on the exams.



Chapter 1 Glossary

MA387 (Real Analysis) Course Information » Glossary » Chapter 1 Glossary

Terms

Cartesian product

Given sets A and B, the Cartesian product of A and B is the set A x B of all ordered pairs (a, b) such that a $\in$ A and b $\in$ B.

Cauchy Convergence Criterion for Sequences

A sequence (a_n) in $\mathbb{R}$ converges iff $\forall$ ε > 0 $\exists$ N $\in$ $\mathbb{N}$ such that n, m $\geq$ N $\Rightarrow$ | $a_n - a_m$ | < ε .

Complete

There are two definitions. First: The set $\mathbb{R}$, constructed by means of Dedekind cuts, is complete if it satisfies the Least Upper Bound Property. Second, $\mathbb{R}$ is complete with respect to Cauchy sequences in the sense that if (a_n) is a sequence of real numbers obeying a Cauchy condition then it converges to a limit in $\mathbb{R}$.

Continuous function

The function $f : [a, b] \to \mathbb{R}$ is continuous if for each ε > 0 and each $x \in [a, b]$ there is a δ > 0 such that $t \in [a, b]$ and $|t - x| < \delta \implies |f(t) - f(x)| < \varepsilon$.

Convergence to a limit

The sequence (a_n) converges to the limit b$\in$ $\mathbb{R}$ as $n \to \infty$ provided that for each ϵ >0 there exists N $\in$ $\mathbb{N}$ such that for all n $\geq$ N, $|a_n - b|$ <ϵ

Figure 2. Screenshot of wiki developed by the class for the second exam.

Assessment

Out of all the usages of the course wiki, the glossary was by far the most unique, and likely the most successful. The key benefits were the following:

- Students received information about what the instructor considered important for upcoming exams;
- Students had an opportunity to simultaneously review and organize key terminology prior to the exam;
- Students were able to correct each other's work.

Students were asked after the first course examination, in which they constructed a glossary, about their feelings on the assignment. The questions asked and responses are shown in Table 1. They were grateful to have the glossaries on the exams, and it

allayed their anxiety somewhat. They liked the collaborative nature of the assignment, and several students submitted entries and beyond the required entries.

	0-15min	**15-30min**	**30-45min**	**45min-60min**	**over 60min**
How much time did you spend working on the glossary?	0	3	4	1	0
	Strongly Agree	**Agree**	**Neutral**	**Disagree**	**Strongly Disagree**
I liked the *collaborative* nature of creating the glossary	2	6	0	0	0
There was enough information provided about editing pages and wiki syntax	3	4	0	1	0
I found mathematical typesetting easier on the course wiki than in Microsoft's Equation Editor	1	2	4	0	1
Adding information to the glossary helped prepare me for the exam	1	6	1	0	0
Looking at the glossary helped prepare me for the exam	1	6	1	0	0
I found the glossary useful during the exam	5	3	0	0	0

Table 1. Results of Student Feedback on the Glossary (Block I)

One of the drawbacks of the glossary was that students tended to make their submissions during the same evening, resulting in some conflicting page requests. Since only one user may edit a page at a time, this forced some students to wait before submitting their entries. However, none of the cadets reported this as a major problem, probably because the small class size made this eventuality rare. For larger classes, it would likely be more of an issue.

Projects

Students were assigned two projects over the course of the semester. The first project required them to read an article in a mathematics journal and write a summary of the article, focusing especially on its connections with the course material. They were also challenged to describe a little of the broader context of the mathematical area addressed by the paper, which is for an undergraduate a particularly challenging task. Rather than turning in a traditional paper report, students submitted their projects on the course wiki by directly editing pages. The first project was intended to provide students practice with reading mathematics, organizing and condensing thoughts, and writing in the mathematical language. After the glossary, this would be one of the first exposures students had to the LaTeX typesetting language.

For the second project, students worked in groups of three to create a wiki page describing a topic in real analysis and an application. The wiki handled the collaborative aspects of the project, with each individual in a group of students able to edit the page. At the time of completion of this paper, students had not yet submitted the assignment, so unfortunately we are unable to speak to this project at the moment.

Assessment

The key positives regarding the wiki approach were the following:

- All edits to pages are logged, allowing the instructor to see exactly when the students are working on the project, and how long they are taking.
- Students had the ability to see each other's projects, giving them plenty of good (and bad) examples to reference; this may have had an additional impact on the quality of their work as well.
- Students provided links to the articles they reviewed directly on the page.
- Some students took advantage of the web format, providing links to mathematicians referenced in the paper as well as sites which could offer the reader additional information about the subject.
- Students who were away from campus on the project due date were able to turn in the assignment as easily as those who remained on campus.

Results of student feedback on the project are shown in Table 1. Some students took advantage of the opportunity to view other projects while they were writing their own, or after they had completed their project. A few students directly mentioned the easy of LaTeX for mathematical typesetting, the ability to see others' work, and the ability to submit the assignment from home. It also seems the assignment offered the students a change-of-pace from usual projects.

	Yes	No	Tried
Did you look at any of the other projects while you were writing your own?	4	5	
Did you look at any of the other projects after you had finished your own?	4	4	1
Did you look at any of the other projects at any time?	7	1	1

1. *In your opinion, what were some of the* ***advantages*** *of submitting the project to the Wiki rather than in a more traditional format?*

1. LaTeX formatting power (3)
2. Unique and different assignment (3)
3. Ability to submit from home (2)
4. Could see others' work (2)
5. Preview capability
6. No unnecessary title pages
7. Later turn-in time

2. *In your opinion, what were some of the* ***disadvantages*** *of submitting the project to the Wiki rather than in a more traditional format?*

1. Learning Curve was steep (5)

2. Problems with formatting… centering, fonts, pictures, etc. (4)
3. Problems with math notation (2)
4. Someone may have lost work

3. *Any other comments about the project?*

- *"There is a steep learning curve with the wiki which added a little bit of work time to the project."*
- *"It takes some time to get to know LaTeX, and how to work with wiki, so I spent more time on that and a little less on the math aspect."*

Table 2. Results of Student Feedback on the Project (Block II)

The primary negatives of the use of the wiki, from the students point-of-view, were the learning curve with regard to notation and wiki formatting. One student cited that some may have lost their work, but did not say that he specifically had. While the learning curve was somewhat of a concern, the primary objective for the first project was reading and writing mathematics. In particular, it was hoped that the students would learn how to use LaTeX, and as a result would have to struggle through it somewhat. In a sense then, the struggles with formatting mathematics were indicators of success. On the other hand, there was a time cost due to using a more unfamiliar markup language for formatting the documents. Some students struggled in particular with uploading and labeling figures.

Conclusion

Discussion of Additional Goals and Concerns

Collaborative Experience

Did the course wiki encourage the students to work more collaboratively? Early student responses indicate that this was the case. Students were especially appreciative of the collaborative nature of the glossary, and appreciated the opportunity, even when they were not working together, to see what other students had done with their projects.

Communication Abilities

This objective is one of the most difficult to assess, as the development of ability to communicate mathematically is such a long-term goal. In addition, much of the improvement in this area is due to more traditional activities such as homework assignments. Thus, the results regarding this objective are inconclusive. The ability of the students to communicate mathematics did improve over the semester, and the course wiki provided one outlet for such communication abilities. But it is difficult to

say whether there was anything inherent in the wiki nature of the course website that worked toward this objective.

Mathematical Typesetting

One of the goals of the project was to enable students to learn LaTeX, the de facto standard in mathematical typesetting. Before the class, most students had experience with Microsoft's Equation Editor and nothing else. All but one student reported that they found it easier to typeset equations using LaTeX than Equation Editor. It is the hope that these students will have an easy transition to typesetting with "true LaTeX" in the future, although this eventuality cannot yet be measured. The objective was highly successful in one case, however, as one of the students began submitting homework assignments in LaTeX towards the end of the semester.

Collaborative Technologies

Students generally liked the wiki approach to the course. There were a few who disliked the wiki in general, but most reported positive experiences with the process. One particularly enthusiastic student developed his own wiki site for his summer AIAD project with another cadet.

Student Participation

While there was low participation on the part of students with optional features (such as the course forum), when grades or bonus points were in play they were quite happy to contribute. There was no trouble with participation in the glossary, for example.

Student Time/Learning Curve

Most students reported that it was not difficult to learn the wiki notation and markup, although there were a few exceptions. The process of adding entries to the glossary took longer than expected, yet was one of the exercises most valued by the students. In the end, there was a time cost to using the wiki on the part of the students, but it was not large and helped them to learn LaTeX, which they would likely learn at a later point anyway.

Instructor Time

There is no doubt that this project required the instructor to learn a lot about wiki sites. However, the time savings created by streamlining the "thought-to-web" pipeline made up for the wiki learning curve, which is not that steep.

Final Comments

In summary, the wiki experiment was very helpful in achieving several of the learning objectives in the course. The glossary project was especially beneficial to the students, and perhaps the element with the greatest impact-to-work ratio. While scaling the glossary idea to larger class sizes could prove to be a challenge, in the small-course environment of the real analysis course it was a greater success than anticipated. While using the wiki to develop/post student projects was also somewhat successful, it is difficult to say for sure based on just one assignment.

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