United States Military Academy

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Master Teacher Program

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Does the Prerequisite Differential Calculus Core Course Offered at the United States Military Academy Adequately Prepare Students for the Follow on Mandatory Calculus-Based Introductory Physics Program?

by

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ABSTRACT

Students at the United States Military Academy are mandated to take differential calculus and a calculus-based physics course as part of their graduation requirement. PH201 is the first course of a two-semester, calculus-based physics sequence. This course consists of an introduction to nuclear physics and a comprehensive study of classical mechanics, which is designed to promote scientific literacy and to develop the use of scientific modes of thought to solve complex problems. This paper discusses the findings of a qualitative analysis derived from student survey data dovetailed with student performance on graded examinations to measure the adequacy in the MA104 Differential Calculus course in preparing students for their future physics PH201 course.

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I. INTRODUCTION

A. BACKGROUND

All students at the United States Military Academy are required to take Differential Calculus, MA104, as the second course in a four course core mathematics sequence. The course is validated by a small population; however, those students who validate the course must then take an advanced level calculus course with differential equations. The MA104 course is offered to second semester 1st year students, and is argued to be a prerequisite course for a multitude of courses offered to 2nd through 4th year students. All students are also required to take Physics, PH201, as another graduation requirement at the academy. PH201 is a calculus-based physics course, and is the first course that students face that depends upon their body of knowledge learned from their MA104 calculus course. This paper discusses the findings of a qualitative analysis derived from student survey data dovetailed with student performance on graded examinations to measure the adequacy in the MA104 Differential Calculus course in preparing students for their future physics PH201 course.

1. MA104: Differential Calculus

MA104 is the second course in a four course core mathematics sequence offered at the United States Military Academy. This course covers the modeling, solution, and discussion of applied problems with the use of single and multi-variable differential calculus.[1] At the time of the study, the course was organized into four blocks of instruction as follows:[2]

Block 1 - Limits, Continuity, and Single Variability Differentiation: Understand the derivative as an instantaneous rate of change and know the graphical, algebraic, and physical interpretation of the derivative.

Block 2 - Problem Solving with Derivatives: Be able to apply the concept of the derivative as an instantaneous rate of change in order to model and solve problems.

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[1] Department of Mathematical Sciences, *MA104 Online Syllabus*, retrieved 21 March 2008 from the World Wide Web at http://www.dean.usma.edu/departments/math/courses/ma104/.

[2] Ibid.

Block 3 - Vector Functions and the Geometry of Space: Use vectors and vector functions to model and solve problems involving objects in space.

Block 4 - Problem Solving with Partial Derivatives: Be able to apply the concept of partial derivatives in order to model and solve problems involving multivariate functions.

At this time, we should note that MA104 is not solely based to prepare students for PH201, but for an array of hard science and engineering curriculums offered at the academy. The results of this analysis should only be interpreted as an indicator of the student's preparedness to enter their physics course.

2. PH201: Introduction to Classical Physics

This is the first course of a two-semester, calculus-based physics sequence. Students take this course immediately after completing their differential calculus course. This course consists of an introduction to nuclear physics and a comprehensive study of classical mechanics, which is designed to promote scientific literacy and to develop the use of scientific modes of thought to solve complex problems. Topics include a survey of nuclear physics and a detailed study of the laws of motion, conservation of energy, and conservation of momentum. An integrated laboratory program illustrates basic scientific techniques and serves to stimulate intellectual curiosity. The core physics program is designed to demonstrate the relevance of physics to military technology and to help prepare future Army leaders to anticipate and adapt to technological change.[3]

B. SCOPE AND PROBLEM STATEMENT

There exist countless questions regarding how to measure a program's suitability to prepare students for future courses. For the purpose of this paper we used two student surveys focusing on the student's compression of 16 Differential Calculus lesson objectives that are applied directly to future Physics course block objectives. In addition, an evaluation of student performance on graded examinations was also used to support the findings derived by student surveys. Lastly we reviewed end of course feedback

[3] Department of Physics, *PH201 Course Details*, retrieved 21 March 2008 from the World Wide Web at http://www.dean.usma.edu/sebpublic/curriccat/crse_details.cfm?crse_nbr=PH201&int_crse_eff_acad_yr=2007&int_crse_eff_term=1

provided by students. This feedback was the result of student free responses to three questions regarding MA104 course improvement.

To complete this paper within the allotted time, and with limited reasonable exploration, the following research questions scope the direction of this research:

- Do the specified 16 lesson objectives found in the prerequisite differential calculus core course (MA104) offered at the United States Military Academy adequately prepare students to understand specific block objectives found in their future calculus-based physics program?
- Do the five specific classroom activities performed in the differential calculus course assist in helping students adequately understand general topics and help students growth in their long term memory of such topics?

II. METHODOLOGY

This chapter outlines the methodology which supports, and bridges, the survey study development to the data analysis. There were three surveys in this study. Students enrolled in both MA104 and PH201 were asked to participate in the survey. Specific values of *n* students for each survey are discussed further in this chapter. The first survey was performed once at the end of the 2007 spring MA104 semester, and the second survey was given in three parts after each block of instruction in the 2001 fall PH201 semester. This chapter also describes the statistical tests used and the measurements chosen to analyze and quantify the concussions.

A. MA104 CALCULUS SURVEY DESCRIPTION

The MA104 calculus survey consisted of 21 questions, and was conducted in one sitting at the end of MA104. The first 16 questions were related to 16 lesson objectives. The students were asked to read each lesson objective, and answer how well they felt that they understood these specific concepts after now completing the MA104 differential calculus course. They were asked to respond via a Likert scale from 1 through 5 (1-being no understanding up to 5-being complete understanding). Each lesson objective was chosen in the survey because of its direct application to block objectives faced in the student's following physics course. These lesson objectives are only a subset of all the lesson objectives for the entire course.

Similarly, the last five questions on the survey asked the students if specific general classroom activities helped increase their understanding of the previous 16 lesson objectives. The size of the population that took the survey was 200 out of the 961 students enrolled in the course. The students were selected for the survey because they were either enrolled in the primary author's class, or they were enrolled within the team of instructor's classes under the leadership of the primary author. Refer to Appendix A to see the full MA104 survey. This survey was completed at the end of the spring 2007 Semester.

B. PH201 PHYSICS SURVEY DESCRIPTION

The PH201 survey consisted of a total of 16 questions and was conducted in three sittings during the following fall 2007 semester. The first 16 questions were related to the same 16 lesson learning objective questions posed in the MA104 survey. The students were asked to read each MA104 lesson objective, and answer how well they thought that their pre-requisite differential calculus course, had prepared them in applying those mathematical constructs in their current PH201 calculus-based physics course. The first portion of the survey was administered following instruction regarding radioactive decay. The second portion was administered following instruction on two-dimensional kinematics and Newton's force laws. The third and final portion was administered after topics of work, torque, angular momentum, and oscillatory motion had been discussed. Students were asked to respond with the same Likert scale (1 through 5) as seen in the previous survey (1, meaning zero transfer of knowledge from MA104 to PH201, up to 5, meaning they thought that MA104 had fully prepared them to apply those calculus concepts in PH201). The size of the population that took the survey was 47 out of the 961 students enrolled in the course. These students were selected because they were enrolled in this papers secondary author's class. Refer to Appendix B to see the PH201 survey.

C. STATISTICAL MEASUREMENT TESTS USED

1. Two Sample *t*-Test

The two sample *t*-test is on of the most commonly used hypothesis tests and can be applied to compare whether the average difference between two groups is truly significant or not.[4] This test was used for this study to compare the averages of the student survey data (refer to Table 1). The analysis showed some interesting results even though the two-sample *t*-test assumes normality, and the survey yields ordinal answers on a Likert scale valued between 1 and 5. As such, a nonparametric test was also conducted to compare findings from the analysis.

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[4] Chew Jian Chieh, *Making Sense of the Two-Sample T-Test, Six Sigma Europe*, retrieved 1 April 2008 from the World Wide Web at http://europe.isixsigma.com/library/content/c070613a.asp.

Null hypothesis: $H_0 : \overline{x}_{physics} - \overline{x}_{calculs} = 0$
Test statistic value: $t = \dfrac{\overline{x}_{physics} - \overline{x}_{calculus}}{\sqrt{\dfrac{\sigma_{physics}{}^2}{m} + \dfrac{\sigma_{math}{}^2}{n}}}$
Alternative hypothesis: $H_a : \overline{x}_{physics} > \overline{x}_{calculs}$

Table 1. Two Sample t-Test

2. Mann-Whitney Test

The Mann-Whitney test is a nonparametric hypothesis test. This nonparametric hypothesis test determines whether two populations have the same population median ($\tilde{\mu}$). It tests the null hypothesis that the two population medians are equal (H$_0$: $\tilde{\mu}_1 = \tilde{\mu}_2$). The alternative hypothesis can be left-tailed ($\tilde{\mu}_1 < \tilde{\mu}_2$), right-tailed ($\tilde{\mu}_1 > \tilde{\mu}_2$), or two-tailed ($\tilde{\mu}_1 \neq \tilde{\mu}_2$).[5] The Mann-Whitney test does not require the data to come from normally distributed populations, but it does need the following assumptions:

· The populations of interest have the same shape.

· The populations are independent.

The Mann-Whitney test uses the ranks of the sample data, instead of their specific values, to detect statistical significance.[6] For this analysis, the null hypothesis is right-tailed. This implies that the survey data provides evidence that the physics median is greater than that of the calculus median. This supports the belief that the calculus course prepares students for their physics course.

Null hypothesis: $H_0 : \tilde{\mu}_{physics} - \tilde{\mu}_{calculs} = 0$
Alternative hypothesis: $H_a : \tilde{\mu}_{physics} > \tilde{\mu}_{calculs}$

Table 2. Mann-Whitney Hypothesis Test

[5] Minitab 15 statistical software package, *Mann Whitney Nonparametric Analysis Overview*, (LEAD Technologies, Inc 2006), help index.

[6] Ibid.

It is important to note, that since the sum of the ranks in the combined sample associated with all observations has a discrete probability distribution, there will not always exist a critical value corresponding to exactly one of the usual levels of significance.[7]

[7] Jay L. Devore, *Probability and Statistics for Engineering and the Sciences, 6th Edition*, Thomson Brooks/Cole, 2004, pgs 678-679.

III. DATA ANALYSIS

This chapter contains the significant results of the data analysis. Within the chapter, there are three sections: Data Compilation, Initial Observations, and Secondary Observations Related to Study Questions. Each section describes the iterative process identifying significant findings.

A. DATA COMPILATION

Receiving a multitude of data consisting of 247 responses to 16 questions, 200 responses to an additional 5 questions, and 872 free responses to three questions regarding course improvement begs the question, what now? The reader should understand that the students selected their answers to the survey biased off of their perceived understanding of each lesson objective. This introduces some variability in the data for both surveys. In addition students answered the physics survey questions one semester after completing their calculus course. As such, a student's memory of certain topics dovetailed with his/her struggling to comprehend applied topics possible introduces more variability in answers to the physics survey. To help answer the questions outlined in the scope of this study, we might hope that the analysis indicates that MA104 course material adequately prepares students for PH201.

B. INITIAL OBSERVATIONS

After compiling the survey data results and looking strictly at the averages for the response to each question, we observe response averages ranging from 3.21 to 4.43 for each calculus survey question. Recall that the response Likert scale had discrete values ranging from 1, meaning no understanding of the lesson objective up to 5, meaning complete understanding of the lesson objective. The higher averages suggest that students perceived that they gained knowledge of the specified lesson objectives and that the course adequately prepared them for their future courses.

Similarly, averages were observed ranging from 3.56 to 4.45 for each physics survey question on the Likert scale from 1, meaning zero transfer of knowledge from MA104 to PH201, up to 5, meaning they thought that MA104 had fully prepared them to apply that calculus concept in PH201. Again, the higher averages suggest that students perceived that the calculus course may have prepared them to apply these specific concepts in their present physics course. The next question becomes, how significant are these findings.

C. SECONDARY OBSERVATIONS RELATED TO STUDY QUESTIONS

The secondary observations help define the significance in the average scores between both surveys, as each survey relates one course to another. In addition, the averages are compared to grades on selected questions from MA104 exams. The two sample *t*-test and the Mann-Whitney test assisted in the hypothesis testing for this analysis.

1. Two Sample *t*-Test Observations

Using the two sample *t*-test we observe that on average, students felt less confident in understanding six MA104 lesson objectives preparing them for physics during the time of their physics survey then they did at the time of taking their calculus survey. This is supported by the fact that $\overline{x}_{physics} - \overline{x}_{math} < 0$ is true with regards to each of these six lesson objectives, with *p*-values well over 50%, not supporting the null hypothesis. Recall that we hope to observe small *p*-values for this test in order to support the null hypothesis. The following points describe the six calculus lesson objectives. It is interesting to note that the first five of these lesson objectives were applied directly in the physics block 1 course objective addressing radioactive decay. The last point was applied in the 6th physics block addressing rigid body dynamics.

- Model using rates of change
- Approximate solutions to models involving rates of change numerically
- Approximate solutions to models involving rates of change graphically
- Understand selected applications of models
- Involving rates of change
- Understanding the definition of the cross product and what it give us

The cross product was covered concurrently with other objectives in a single day. As the semester progressed, students relied on mathematical Computer Algebra System (CAS) to compute the cross product. The calculus course did not mandate the manual computation of the cross product, and focused mainly on the physical interpretation of what the cross product yielded – an orthogonal vector in relationship to the two crossed vectors in a plane.

In addition, student opinion captured on the free response MA104 end of course survey (refer to Appendix D) shows that students recommend adding more time on topics such as related rates and vectors to improve the mathematics program. These responses (refer to Appendix D) support observations found using the two sample *t*-test noted above with regards to rates of change and understanding the cross product.

With regards to the 11 remaining survey questions, the two sample *t*-test indicates that students appeared more confident in their understanding of these lesson objectives preparing them for physics during the time of the physics survey then they did at the time of taking the calculus survey. This finding supports with a small level of significance that the calculus course did prepare the students for physics. However, the measurability of this insight is again difficult, as the results of the survey are based on perceived gained knowledge, coupled with the possibility that seeing the topics a second time during physics applications skewed what they thought they remembered understanding at the time of their calculus course.

There exist however, 3 out of the eleven lesson objectives in this category where the *t*-statistic yielded a p-value < 0.05, and one lesson objective where a *t*-statistic yielded a p-value close to 0.1. These extremely low p-values indicate a strong significance that the calculus course did adequately prepare the students for their current physics course block objectives. These 4 lesson objectives are as follows.

- Develop vector functions that describe the motion of an object through space
- Understand vector addition, subtraction, and scalar multiplication algebraically and graphically
- Understand the definition of the dot product and how it can be computed
- Understand what it means for two vectors to be orthogonal

These specific lesson objectives appeared in MA104 not only during introductory lessons, but they also appeared in applications throughout the calculus course. These same objectives appeared in the 2nd, 3rd, and 4th physics blocks addressing kinematics, Newton's Law, and conservation of energy.

2. Mann-Whitney Test Observations

The medians from the Mann-Whitney test may provide additional insight to the analysis as the discrete Likert scale from 1 to 5 may not have strictly met the assumptions necessary for the two sample *t*-test. With regards to each survey question, we would hope that the test yielded a strong test statistic such that the ranked ordered median value for survey answers increase from the time of the calculus survey to the time during the physics survey.

Even though the six areas where students felt less comfortable understanding the calculus lesson objectives as they were applied to the physics course as indicated by previously stated extremely high *p*-values, the ranked median for both surveys was still the same. This value was a 4 on the scale from 1 to 5. In addition, the ranked median was identical with regards to four of the same 11 remaining survey questions outlined in the two sample *t*-test observations. In fact, the ranked median was identical for 13 out of the 16 compared survey questions. We had hoped that the medians in the physics survey were higher than the medians in the calculus survey providing a strong significance that MA104 adequately prepared students for their follow-on physics course. The equality in the ranked medians for these compared survey questions does not contradict our hopes. It merely says that we cannot conclude that these lesson objectives helped the students.

Additionally, a significantly low *p*-value existed with respect to several topics when comparing survey results. Each was calculated from the Mann-Whitney test for both understanding the graphical and physical interpretation of the derivative in terms of vector functions; and the relationship between the position, velocity, and acceleration vectors for an object's motion in space. This suggests that these two calculus objectives sufficiently prepared students for kinematic applications in their physics course. This is interesting because though the *p*-value is low, additional analysis discussed in section four of this chapter indicates low performance on MA104 exams regarding both of these lesson objectives.

Additionally the value of the median dropped for three compared survey questions. This drop in the median requires further investigation. Outlined at the end of this paragraph are the three lesson objectives. Surprisingly, understanding what it means for two vectors to be orthogonal appeared troublesome with this test even thought it previously had a significantly low *p*-value when evaluated with the two sample *t*-test. Insight from the free response MA104 survey supported student's wishes, as measured by the Mann-Whitney test, to spend more time on evaluating derivatives of basic algebraic functions. We observe in Appendix D multiple comments from students suggesting a desire to spend more time on evaluating basic derivatives.

- Evaluating derivatives of basic algebraic functions
- Understand what it means for two vectors to be orthogonal
- Apply the dot product to find the work done by a force

These calculus lesson objectives are applied to the physics block objectives, addressing kinematics and conservation of energy. The dropped value in each ranked median does raise some concern. The lowered median is possibly explained by the following. It is observed in MA104, that students learn to evaluate derivatives by hand at the beginning of the course, but soon rely heavily on CAS to continue evaluating derivatives as the course continues. Similarly, students eventually rely on CAS to calculate the dot product between two vectors. The lack of performing it by hand, or graphically displaying the vectors probably has some correlation to the dropped confidence in applying the dot product in applications of work when the students are then forced to compute applications by hand after learning and reinforcing that learning using technology. The computation of the dot product is currently taught at the same time as the previously mentioned cross product. In addition, the modeling aspect of applied problems in the calculus course tends to move into complicated problems where solutions benefit from the use of CAS.

3. Analysis on Survey Questions Pertaining to Classroom Techniques During Problem Solving Laboratories (PSLs)

As mentioned in the MA104 Calculus Survey Description portion of this paper, there were five questions asking calculus students if certain classroom techniques assisted in their understanding of the 16 specified lesson objectives. The original sample size (n) of the MA104 survey included students who were taught by both the primary author of this paper, as well as other instructors. As not all instructors taught the same way, the sample size of students for this portion of the analysis is reduced to only those who were under the instruction of the primary author. As such, n = 48 for this portion of the analysis. The five questions outlined in Figure 1 are based off the five techniques used during separate classroom problem solving laboratories performed throughout the semester.

MA104 Calculus Additional Survey Questions		Average	Mode
1. Did playing Games (IE Jeopardy) increase your knowledge of any of the above mentioned content?		3.09	4
2. Did working individually during Problem Solving Labs increase your knowledge in any of the above mentioned areas?		3.31	4
3. Did working in small groups during Problem Solving Labs increase your knowledge in any of the above mentioned areas?		3.75	4
4. Did working as one large group in class during Problem Solving Labs increase your knowledge in any of the above mentioned areas?		3.31	4
5. Did the Projectile Motion Lab with the Artillery Simulator increase your knowledge in any of the above mentioned areas?		3.94	4

Table 3. MA104 Calculus Additional Survey Questions Pertaining to PSL Techniques

Table 3 displays the averages and modes to each of these five MA104 survey questions. Students rated the usefulness of each technique in assisting their learning the previously mentioned lesson objectives. A Likert scale of 1 to 5 was used again (1, being did not assist in understanding the lesson objectives up to 5, being completely assisted in the understanding of the lesson objectives). Both the average and mode depict righty skewed results for each survey. As such, there is an inclination that each of these classroom exercises did assist in the gained knowledge of course material.

Though all techniques were deemed helpful by the students, we observe from Table 3 that the lowest average pertains to playing games in the classroom. Hence students favored the other techniques over playing games. This is an interesting finding, as the students often seemed to respond favorable when playing the games reinforcing topics. Applying a two sample t-test we observe the following results outlined in table 4.

	working individualy vs playing games	working in small groups vs playing games	working as one large group vs playing games	projectile motion lab vs playing games
t -test statistic	1.034265	3.100633	1.053771	3.978609
p -value	0.15	0.001	0.14	0.00006

Table 4. Students Preference of Classroom Techniques Over Playing Games

The table in Figure 4 indicates that working individually, working in small groups, working as one large group, and performing the projectile motion laboratory were all preferred over playing games in the classroom when trying to reinforce the 16 lesson objectives. In addition, the extremely low *p*-values suggest that students rated working in small groups and performing the projectile motion laboratory as the two best techniques assisting in their understanding of course material.

Additionally, multiple free responses to the MA104 end of course survey (refer to Appendix D) comment on the effectiveness of the Problem Solving Labs. Comments to improve the course suggest adding more labs, as they help student growth in understanding material. Comments to sustain the mathematics program equally suggest maintaining the labs, as students claim that they benefit from each lab as they are applicable to real world situations. One aspect not covered in the statistical analysis is board work. However, we observe in Appendix D that students favor being "sent to the boards" to work mathematical problems. This technique of evaluating student performance is much appreciated by the students, and should be integrated into the mathematics curriculum as much as possible to include during labs. We also observe from the free response question presented in Appendix D that the students viewed relevancy to the curriculum towards the real world as useful. This was seen in multiple responses, as the students commented positively towards the relevant topics discussed during the Problem Solving Labs.

4. Student Performance on Graded Examinations

The MA104 course had a single exam after each of the four blocks of material and an additional Final Exam - totaling five exams. As mentioned in the introduction, MA104 does not only prepare students for PH201, but also for an array of hard science and engineering curriculums offered at the Academy. As such, only three of the five MA104 examinations are considered for the purpose of this survey. The examinations include: Exam 2 - Problem Solving with Derivatives; Exam 3 - Vector Functions and the Geometry of Space; and the Final Exam.

Both Exam 2 and the Final Exam support the Mann-Whitney test results suggesting that students felt underprepared when tested on the evaluation of derivatives of basic algebraic functions. The student average of 82.21% on Exam 2, coupled with the lowered average on the Final Exam of 78.84% (refer to Appendix C), concerns the authors. Though both of these scores are well above the typical average score of 70%, the mathematics department requires students to receive at least a 80% on this portion of Exam 2 in order to pass the course. During the spring of 2007, if a student passed this portion of the second exam, he/she received a GO, however, if they receive less than 80% during this exam, they receive a NO GO, requiring additional instruction on basic derivatives, as well as the need to continue taking additional exams until receiving a passing score of greater than 80%. The lowered average to 78.84% (refer to Appendix C) for the same topic on the Final Exam supports the earlier observation that as soon as students received a passing score on taking derivatives by hand, they quickly forget how to solve derivatives by hand as students then heavily relied on CAS for the remaining of the semester.

Similarly, students' scores dropped on the MA104 Final Exam when compared to their 2nd MA104 exam performance with respect to modeling with rates of change. This suggests that the long term memory in applying this topic to problems is not as strong as it could be. The declined in test scores also supports the extremely large *p*-values found with the two sample *t*-test suggesting that students were not as prepared for the application of this topic in physics.

In addition, we observe lower exam averages with regards to the MA104 lesson objective concerning applications of vector functions as outlined by the MA104 survey questions numbers 6 and 8. This finding does not necessarily support the results as suggested by the Mann-Whitney test, as each of the medians were the same, however, it is supported by the higher *p*-values found from the two sample *t*-test. Looking at the exam scores adds the additional insight that was needed after viewing the hypothesis test results. From the final exam, we again see that the students were not as prepared for the applications of these topics in physics.

On a positive note, one area where test scores support the survey analysis as well as helps provide additional insight is with regards to the dot product. The high average score of 95% on the final exam questions (refer to Appendix C) with respect to computing the dot product as well as what it means for two vectors to be orthogonal does support the survey data. We are certain that the student's comfort level in computing the dot product was very high during both his/her calculus course and physics course.

Other averages on the exams are presented in Appendix C for the reader to examine. No further tests were conducted to see if a correlation exists between the test scores and the survey data. However, it should be noted that students felt "that the exams really made sure that [they] understood the concepts" (refer to Appendix D).

VI. CONCLUSIONS AND RECOMMENDATIONS

This chapter contains a summary of conclusions and gained insight from the data analysis. Following the summary of conclusions and gained insight section of this chapter are some recommendations for future study.

A. SUMMARY OF CONCLUSIONS AND GAINED INSIGHT

The summary of conclusions and gained insight has two sections: Survey Data Analysis Conclusions and Future MA104 Development.

1. Survey Data Analysis Conclusions

This section attempts to answer the scoped questions provided earlier in this paper. Both questions are restated in this section prior to their conclusions.

Do the specified 16 lesson objectives found in the prerequisite differential calculus core course offered at the United States Military Academy adequately prepare students to understand specific block objectives found in their future calculus-based physics program?

We see a mixture of responses on the survey pertaining to each question. Looking strictly at the averages to each questions response, we can say that the specific 16 lesson objectives do adequately prepare students for physics. Digging a bit deeper and performing the hypothesis testing, we observe a mixture of outcomes as well as levels of significance for each of the lesson objectives as mentioned in the Data Analysis portion of this paper. Some significantly suggest that the students mastered the lesson objectives and where able to apply them confidently to their physics course. Specifically, the four lesson objectives outlined at the end of this paragraph are well received by the students, and students feel as though they were able to easily apply them to their physics course.

- Develop vector functions that describe the motion of an object through space
- Understand vector addition, subtraction, and scalar multiplication algebraically and graphically
- Understand the definition of the dot product and how it can be computed
- Understand what it means for two vectors to be orthogonal

In addition, there are results of the hypothesis testing coupled with some analysis of the exam scores that provide interesting findings. These findings do suggest that improvements could be made to the MA104 curriculum to assist the students learn several MA104 lesson objectives needed for applications in their physics course.

For example, the students understood the computation of the dot product between two vectors; however, they were uncomfortable with applying the dot product to applications such as the work done upon an object. Student comfort levels when modeling with rates of change, applying the cross product, evaluating derivatives of basic algebraic functions without CAS, and vector functions appeared to be less then desired. As such, several suggestions are mentioned later in this chapter entitled Future MA104 Development.

Do the five specific classroom activities performed in the differential calculus course assist in helping students adequately understand general topics and help student growth in their long term memory of such topics?

Yes, the survey data indicates that Problem Solving Labs (PSLs) benefit the students' learning of these 16 calculus lesson objectives needed for their physics course. The data averages and modes favor the PSL techniques. However, it is interesting to note that the technique of playing games was least favorable to the students when compared to the other techniques used. In addition, students favored working in small groups and working on specific real world problems such as the projectile motion laboratory. This again is reinforced by the student's free responses to the end of course survey commenting on the effectiveness of the PSLs addressing real world applications. The biggest take away here is that real world applications working in small groups facilitate long term remembrance of specified learning objectives.

2. Future MA104 Development

Changes in the MA104 calculus course to the Spring 2008 semester that were supported by the outcome of this analysis included adding an additional PSL to cover topics in modeling using rates of change, adding the cross product to the end of block exam, and recommending the testing of the cross product during the final exam.

Changes in the MA104 calculus course for the Fall 2008 semester that are supported by the outcome of this analysis include.

- Adding additional instructional time covering introductory lessons on vectors and applying vectors to solving basic kinematics and conservation of energy type problems. There will be two entire lessons dedicated to this topic, where as currently there is only one lesson. Spending an additional day on the basics and computing solutions by hand are expected to pay dividends when solving complicated problems via the computer in follow on application lesson.
- Adding additional instructional time covering the dot product in an introductory lesson. There will be one entire lesson dedicated to the understanding the dot product, how to compute it, what the result of the dot product produces. Only after this will we examine applications of the dot product more thoroughly in follow on lessons. Currently, the dot product is examined in half a lesson taught concurrently with the cross product. Though computation of the dot product appeared favorable, the applications of it are often confused as students are not comfortable deciding whether to apply the dot product or cross product to real world applications.
- Adding additional instructional time covering the cross product in an introductory lesson. There will be one entire lesson dedicated to the understanding the cross product, how to compute it, and what the result of the cross product produces. After this lesson, we will examine applications of the cross product in follow on lessons. Currently, the cross product is examined in half a lesson taught concurrently with the dot product.

Recommended changes in the MA104 calculus course for Fall 2008 that have not been approved yet, but are supported by the outcome of this analysis include.

- Adding additional Problem Solving Labs (PSL). The survey results indicate that each PSL vastly contributed to understanding complicated applications of previously taught material. In addition, instructors will be highly encouraged to change the classroom techniques for each PSL, while minimizing the technique of playing games. Changing the classroom technique for each PSL will benefit the diversity of learning strategies for each student.

- Creating a course wide quiz for all students to take at the same time in their curriculum covering the evaluation of derivatives of basic algebraic functions without the use of CAS. By creating a course wide quiz, we hope that the students may study specifically for this topic rather than possible lightly brushing over while studying for a larger exam covering a multitude of applications. We hope that the specified studying for this topic will assist in the long term remembrance in performing the differentiation rules by hand.

B. RECOMMENDATIONS FOR FUTURE STUDY

As the research unfolded, a multitude of tangential and parallel topics came into light for future study. One particular area of study is to compare and contrast the data analysis output from this project to similar studies over the course of several years. The following suggestions specify questions that would add future insight while using this current analysis as a preliminary literature review.

- Perform a similar study after implementing changes to the MA104 course development, and see if changes affect the student's perceived subject knowledge.
- How is Computer Algebra System (CAS) affecting the long term computational behavior of students in engineering and science based curriculums?
- How is CAS affecting student's long term cognitive mathematical body of knowledge?
- How is student use of CAS being leveraged by client disciplines, e.g., physics?

APPENDIX A. MA104 SURVEY

Appendix A and Appendix B are copies of the MA104 and PH201 surveys provided to selected students. The MA104 survey was conducted on a piece of paper with the identical format shown in this appendix. The PH201 survey was conducted with the use of classroom digital software. The graphical user interface (GUI) seen by the students is shown in Appendix B. Though the format in Appendix B appears different than seen in Appendix A, the surveys were similar in nature as they both asked the students to comment on how well they thought MA104 prepared them on the numbered lesson objectives.

Numbered below are MA104 lesson objectives. Each are listed beneath six PH201 Block objectives. The MA104 lesson objectives are calculus prerequisite skills required to discuss and understand applications in the physics (PH201) course you will be taking next fall. Read each lesson objective, and answer how well you feel that you understand the objectives after now taking MA104.

Answer 1 through 5 (1-being no understanding up to 5-being complete understanding)

	Physics Block 1: Addressing Radioactive Decay					
1.	Model using rates of change	1	2	3	4	5
2.	Approximate solutions to models involving rates of change numerically	1	2	3	4	5
3.	Approximate solutions to models involving rates of change graphically	1	2	3	4	5
4.	Understand selected applications of models involving rates of change	1	2	3	4	5
	Physics Block 2: Addressing Kinematics					
5.	Evaluate derivatives of basic algebraic functions	1	2	3	4	5
6.	Develop vector functions that describe the motion of an object through space.	1	2	3	4	5
7.	Know the graphical and physical interpretation of the derivative in terms of vector functions.	1	2	3	4	5
8.	Understand the relationship between the position, velocity, and acceleration vectors for an object’s motion in space	1	2	3	4	5

Physics Block 3: Addressing Newton's Laws					
9. Understand what a vector is and the properties of Vectors	1	2	3	4	5
10. Understand vector addition, subtraction, and scalar multiplication algebraically and graphically	1	2	3	4	5
11. Understand how a vector can be used to describe several forces acting on an object, and how the resultant force is the sum of these forces	1	2	3	4	5
Physics Block 4: Addressing Conservation of Energy					
12. Understand the definition of the dot product and how it can be computed	1	2	3	4	5
13. Understand what it means for two vectors to be Orthogonal	1	2	3	4	5
14. Apply the dot product to find the work done by a force	1	2	3	4	5
Physics Block 6: Addressing Rigid Body Dynamics					
15. Understand the definition of the cross product and what it gives us	1	2	3	4	5
Physics Block 7: Addressing Applications (Simple Harmonic Motion)					
16. Classify all critical numbers and endpoints as having a local or absolute maximum or minimum value or neither	1	2	3	4	5
General Questions:					
17. Did playing Games (IE Jeopardy) increase your knowledge of any of the above mentioned content?	1	2	3	4	5
18. Did working individually during Problem Solving Labs increase your knowledge in any of the above Mentioned areas?	1	2	3	4	5
19. Did working in small groups during Problem Solving Labs increase your knowledge in any of the above mentioned areas?	1	2	3	4	5
20. Did working as one large group in class during Problem Solving Labs increase your knowledge in any of the above mentioned areas?	1	2	3	4	5
21. Did the Projectile Motion Lab with the Artillery Simulator increase your knowledge in any of the above mentioned areas?	1	2	3	4	5

APPENDIX B. PH201 SURVEY

Part 1

1.) Model using rates of change

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

2.) Approximate solutions to models involving rates of change numerically

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

3.) Approximate solutions to models involving rates of change graphically

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

4.) Understand selected applications of models involving rates of change

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

Part 2

1.) Evaluate derivatives of basic algebraic functions

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

2.) Develop vector functions that describe the motion of an object through space

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

3.) Know the graphical and physical interpretation of the derivative in terms of vector functions

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

4.) Understand the relationship between the position, velocity, and acceleration vector's for and object's motion in space

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)

Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

5.) Understand what a vector is and the properties of vectors

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

6.) Understand vector addition, subtraction, and scalar multiplication algebraically and graphically

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

7.) Understand how a vector can be used to describe several forces acting on an object, and how the resultant force is the sum of these forces

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)

Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

Part 3

1.) Understand the definition of the dot

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

2.) Understand what it means for two vectors to be orthogonal

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

3.) Apply the dot product to find the work done by a force

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

4.) Understand the definition of the cross product and what it gives us

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

6.) Classify all critical numbers and endpoints as having a local or absolute

Did not prepare for PH201 (strongly)
Did not prepare for PH201 (weakly)
Neutral
Did prepare for PH201 (weakly)
Did prepare for PH201 (strongly)

APPENDIX C. EXAM QUESTIONS SUPPORTING MA104 CALCULUS SURVEY

This appendix provides three tables corresponding to the MA104 calculus student examinations. Each question is linked to the surveys used in gaining student feedback.

Exam Question Supporting Survey	Lesson Objective Linked to MA104 Survey Question	Student Average %	Student Standard Deviation
1	5. Evaluate derivatives of basic algebraic functions	82.81	16.85
2	1. Model using rates of change 4. Understand selected applications of models involving rates of change	85.05	15.58

Table 5. MA104 Exam 2 - Problem Solving with Derivatives

Exam Question Supporting Survey	Lesson Objective Linked to MA104 Survey Question	Student Average %	Student Standard Deviation
1	7. Know the graphical and physical interpretation of the derivative in terms of vector functions 8. Understand the relationship between the position, velocity, and acceleration vectors for an object's motion in space 9. Understand what a vector is and the properties of vectors	79.46	18.56
2	6. Develop vector function that describe the motion of an object through space 9. Understand what a vector is and the properties of vectors	86.81	14.83
3	12. Understand the definition of the dot product and how it can be computed 13. Understand that it means for two vectors to be orthogonal	88.16	16.36
4	12. Understand the definition of the dot product and how it can be computed	94.83	13.27
6	8. Understand the relationship between the position, velocity, and acceleration vectors for an object's motion in space	81.84	22.1
7	6. Develop vector function that describe the motion of an object through space 8. Understand the relationship between the position, velocity, and acceleration vectors for an object's motion in space	67.24	25.35

Table 6. MA104 Exam 3 - Vector Functions and the Geometry of Space

Exam Question Supporting Survey	Lesson Objective Linked to MA104 Survey Question	Student Average %	Student Standard Deviation
1	5. Evaluate derivatives of basic algebraic functions	78.84	11.47
2	6. Develop vector function that describe the motion of an object through space 9. Understand what a vector is and the properties of vectors	82.51	12.52
4	6. Develop vector function that describe the motion of an object through space 7. Know the graphical and physical interpretation of the derivative in terms of vector functions 8. Understand the relationship between the position, velocity, and acceleration vectors for an object's motion in space	83.58	15.51
8	12. Understand the definition of the dot product and how it can be computed	95.04	11.12
9	4. Understand selected application of models involving rates of changes	75.97	24.07
12	6. Develop vector function that describe the motion of an object through space 7. Know the graphical and physical interpretation of the derivative in terms of vector functions 8. Understand the relationship between the position, velocity, and acceleration vectors for an object's motion in space	77.17	20.4

Table 7. MA 104 Final Exam

APPENDIX D. MA104 END OF COURSE STUDENT SURVEY

Appendix D provides a small sample from 872 free responses provided by students when responding to 3 questions. These responses provide supporting insight to the analysis of which conclusions are drawn.

A. QUESTION 1: IF YOU COULD MAKE ONE IMPROVEMENT TO THE MATH PROGRAM, WHAT WOULD IT BE AND WHY?

1. Less focus on using technology and more explanation on hos to perform thigs by hand. Technology is not alwas available or working correctly.
2. I would spend more time on vectors in MA103, so vector calculus is fresher in the mind and the concepts are easier to grasp
3. More Problem Solving Labs. They integrate all the abstract concepts we learn, and allow us to solve complex problems we might approach one day.
4. Have more Problem Solving Labs
5. More Problem Solving Labs because they show a variety of problems that the class can discuss so that those who participate understand the material better.
6. Study less topics, over longer periods of time.
7. I would say to spend a little more time on the rules for derivatives, because I know that I had never had any experience with them before this course and it was hard for me to keep all the rules straight.
8. More time on related rates, it was the toughest part.
9. I would teach derivative rules during the first semester so that students have a more solid base to build off of with concepts.

B. QUESTION 2: IF YOU COULD KEEP ONE ASPECT THE SAME WITHIN THE MATH PROGRAM, WHAT WOULD IT BE AND WHY?

1. The real world applications.

2. Although I hated the exams, the non technology portions of the exams really made sure that I understood the concepts.

3. I would keep student board work because it makes the student realize whether or not he/she really understands the material.

4. I would keep student board work.

C. QUESTION 3: THE CONTENT OF THE MATH CURRICULUM APPEARS/DOES NOT APPEAR RELEVANT TO THE REAL WORLD BECAUSE…

1. The content does appear relevant to the real world, especially when we consider homework, projects, and Problem Solving Labs.

2. It appears to be relevant, especially with respect to projectile motion and the Field Artillery. Calculus is essential though for all sorts of abstract problems. If I need to maximize profit on industrial production for example, I can, given the actual problem.

3. It appears to be relevant because we can use it for things like Field Artillery, should be chose to branch that. It can also be used be economists or those using physics to answer questions about the real world.

4. It appears like when we did the artillery class.

5. We can use it in many aspects of physics.

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