Student Selection for the Advanced Mathematics Program at the U.S. Military Academy

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Abstract

Every year the U.S. Military Academy is faced with selecting highly qualified students to fill the seats in its Advanced Mathematics Program. Traditionally, selection has been accomplished using a tiered performance system; qualified students are placed within five tiers related to chosen performance criteria. These tiers form the basis for student selection. In this paper we will present a new approach to student placement. Using historical student performance and final grade data as well as statistical tools such as multi-regression analysis, mathematical models were created to predict student performance. It will be shown that these models do a very good job of predicting final course grades in the Program. We will also examine results from a program culminating survey and determine that students are fully benefiting from the Program's curriculum and selection process.

Advanced Mathematics Program Overview

The Advanced Mathematics Program (AMP) at the United States Military Academy (USMA) is a two-semester program consisting of courses in multi-variable calculus and differential equations. It is designed for approximately 180 select first year students who have completed a single-variable calculus course (in high school or college) and have the potential to excel at a higher level than the average entering student. The Program's goal is to expose students to advanced concepts in mathematics while leveraging their natural desire to learn. Students enrolled in the Program are exposed to topics beyond the regular core math courses at the USMA, complete a structured technical writing program, and enjoy guest lecturers from the fields of engineering, physics, and computer science. As a result of the AMP nearly 70% of participating students from the USMA Class of 2007 through the Class of 2009 elected a math, science, or engineering (MSE) major. This is at a rate nearly twice that of the general student population at the USMA.

The core of the AMP is the student. Selecting the most qualified students – those who demonstrate the ability to learn at an increased rate – allows the creation of curriculum rich in advanced mathematical concepts. The selection process begins several weeks prior to the start of the fall semester and considers AP Calculus Exam AB (AP-CALC) and BC scores, college single-variable calculus course grades, as well as student performance on the USMA Single-Variable Calculus Validation Exam (USMA VAL).

Selection Process

Each year, the USMA enrolls approximately 1,300 new students. Of these students, approximately 400 have previously taken a course in calculus with about 170 having AP-CALC scores on file at USMA. During the summer prior to the first academic semester, students are administered an open-invitation single-variable calculus validation exam, USMA VAL. While this exam is open to all new students, the majority of pre-exam advertising is aimed at students

who have previously taken a course in calculus. The USMA VAL is 110 minutes long and assesses student comprehension of single-variable calculus concepts (typically topics covered in college calculus I & II courses). This exam is administered to approximately 350 students. After grading the validation exam, eligible students are placed into an appropriate selection tier (Table 1) based on criteria discussed above.

Tier	USMA VAL Score	AP-CALC BC Score	AP-CALC AB Score	College Calculus Grade
1	≥80%	-	-	-
2	≥60% &	≥3 or	≥4 or	A
3	≥40% &	≥3 or	≥4 or	A
4	≥67%	-	-	-
5	≥54%	-	-	-

Table 1: Selection Tiers.

Prior to the start of classes, all selected students receive a letter from the Math Department Head extending them an invitation to join the AMP. It is important to note that enrollment in the Program is voluntary – this is especially important at an institution where much of the students' efforts are mandatory. Nearly all of the students accept this invitation and initial AMP enrollment is historically around 195 students.

The performance of selected students is monitored through the first two weeks of the semester. During this time students acclimate to the pace of the course and instructors assess students' skills through in-class work, as well as at least one graded homework assignment and graded quiz. The culminating event of the Program's selection process is the administration of the Single-Variable Calculus Fundamentals Concepts Exam (FCE) at the end of the second week. While similar to the summer validation exam, there is a much higher performance expectation on this exam as students have had two weeks to prepare for the FCE. An acceptable score on the FCE is considered 80% while the average is historically 87%. At this point in the selection process approximately 10-25 students will normally transfer out of the Program, either by self nomination or by the suggestion of their instructor, resulting in a final enrollment of approximately 180 students (roughly 14% of each entering USMA class).

Performance data collected over the past two academic years (Table 2) permits us to assess the validity of each tier by examining the relationship between selection tier and actual performance in both the multi-variable calculus and differential equations courses.

Tier	USMA VAL Average Score	AP-CALC AB Average Score	Multi-Variable Calculus Average Grade	Differential Equations Average Grade	AMP Average
1	85 %	4.8	93 %	91 %	92 %
2	70 %	4.6	91 %	88 %	90 %
3	52 %	4.3	89 %	86 %	87 %
4	74 %	-	91 %	88 %	89 %
5	58 %	-	91 %	88 %	89 %

Table 2: Selection Tier Performance Data for USMA Classes of 2009 & 2010.

As a validation of the Program's selection criteria, Table 2 shows the expected relationship amongst Tier 1-3 students – final grades decreasing with descending tiers. However, an interesting relationship is found in the average AMP final course grades (AMP Average) for students in Tiers 4 & 5 (again, students in Tiers 4 & 5 either did not take the AP-CALC or did not have scores on record at the USMA). The performance data for Tiers 4 & 5 show that in the absence of AP-CALC scores, the USMA VAL serves as a great indicator of success. Interestingly, there seems to be a near linear relationship between the SAT-M score average and the AMP Average, as shown in Table 3.

Tier	# Students	USMA VAL Average Score	**SAT-M Average Score**	AMP Average
1	70	85 %	**722**	92 %
2	115	70 %	**705**	90 %
3	53	52 %	**682**	87 %
4	54	74 %	**709**	89 %
5	37	58 %	**701**	89 %

Table 3: Selection Tier Performance Data with SAT-M Score for USMA AMP Classes of 2009 & 2010.

SAT-M Score as a Performance Indicator

Historically, the SAT-M score has been used as a means of making qualitative assessments on students that do not have complete sets of performance data, rather than a primary selection criterion. However, the above relationship between SAT-M score and AMP Average points to the question, *should the SAT-M score be considered as a primary criterion for student selection?*

There are a number of studies that examine the use of the SAT-M score in the prediction of grades in college mathematics courses. Bridgeman and Wendler (1989) state that the SAT-M score itself is a relatively poor predictor of success in college mathematics courses when compared to tests specifically designed for placement purposes, but the predictive validity of SAT-M for success in calculus courses is better then it is of algebra/precalculus grades. They conclude that if it can be shown that the SAT-M score is a reasonably good predictor for a particular mathematics course then it may have a role to play in the selection of students. Odell and Schumacher (1995) showed an encouraging relationship: placement tests used in conjunction with SAT-M scores can be a better predictor than SAT scores alone. Their conclusion was based on data from a private business college in Rhode Island. Not all studies found a favorable relationship between SAT-M scores and final grades in mathematics courses. Ruenda and Sokolowski (2004) found that the SAT-M score was not a good predictor and concluded that the local placement exam used did a much better job of predicting student success. Their conclusion was based on data from Merrimack College, a private 4-year Catholic college in Massachusetts, with the context of placing students into mathematics courses ranging from college algebra to Calculus I.

Performance Data Analysis and Results

In order to determine the relationship between SAT-M score and AMP Average we conducted a multiple regression analysis using USMA AMP Class of 2009 & 2010 data. Three models were created to predict AMP Average. The first model is based on SAT-M and USMA VAL and is given by the equation below:

AMP Average = 67.0+ .0237(SAT-M) + .0860(USMA VAL) Model 1

A *t*-test for each of the variables in Model 1 indicated that the SAT-M had a *t*-value of 4.76 (P <.0005) and the USMA VAL had a *t*-value of 4.43 (P <.0005). Here we can see that both performance indicators are significant. Model 1 resulted with a *P-squared* value of 14.9%.

A second model was created to predict AMP Average based on USMA-VAL and AP-CALC. This model accounts for the primary performance criteria that are currently taken into account by the tiered selection process. The second model is given by the equation below:

AMP Average = 72.9 + .0822(USMA VAL) + 2.43(AP-CALC) Model 2

A *t*-test for each of the variables in Model 2 indicated that the USMA VAL had a *t*-value of 3.66 (P <.0005) and the AP-CALC had a *t*-value of 4.26 (P < .0005). We can see that both performance indicators are significant in this model as well. The second model resulted with a *P-squared* value of 17.6%.

A third model was created to predict AMP Average based on SAT-M, USMA-VAL, and AP-CALC. Model 3 is given by the equation below:

AMP Average = 60.5 + .0222(SAT-M) + .0598(USMA VAL) + 2.06(AP-CALC) Model 3

A *t*-test for each of the variables in Model 3 indicated that the SAT-M had a *t*-value of 3.74 (P <.0005), USMA VAL had a *t*-value of 2.64 (P <.0009) and the AP-CALC had a *t*-value of 3.67

($P < .0005$). We can see that all three performance indicators are significant here as well. The third model resulted with a *P-squared* value of 22.3%.

In order to compare the three models we first looked at how well each model predicted USMA AMP Class of 2011 AMP Average. Table 4 show a side-by-side comparison of the three models with historical data.

Tier	AP-CALC Average Score	USMA VAL Average Score	SAT-M Average Score	AMP Average	Model 1: SAT-M & USMA VAL	Model 2: USMA VAL & AP-CALC	Model 3: SAT-M, USMA VAL, & AP-CALC
1	4.9	84 %	729	91.6 %	91.5 %	90.8 %	91.3 %
2	4.7	69 %	705	89.8 %	89.6 %	90.0 %	89.9 %
3	4.3	50 %	674	87.1 %	87.2 %	87.5 %	87.4 %
4	-	72 %	711	88.1 %	90.1 %	-	-
5	-	59 %	694	86.4 %	88.5 %	-	-

Table 4: Prediction Model Tier Comparison for USMA AMP Class of 2011.

Notice that Model 2 and 3 cannot predict Tier 4 or 5 AMP Average since the Academy lacks AP-CALC data on these students. At first glance, it appears all the models do a pretty good job of predicting AMP Average. In order to get a better look at each model's predictions an error analysis was done and is shown in Table 5.

Model	Mean Squared Error	Standard Deviation
1	20.7	4.4
2	19.7	4.4
3	18.7	4.2

Table 5: MSE and Standard Deviation Comparison for USMA AMP Class of 2011.

We can see in Table 5 that the mean squared error (MSE) and standard deviation (σ) are similar for all of the models' predictions. Model 3 seems to be the most accurate, however, Model 1 is noteworthy in that every AMP candidate has taken the USMA VAL and has a SAT-M score (or ACT Math equivalent) on file. We can also see that Model 1 and 2 are similar in the accuracy of their predictions. Since $2(\pm\sigma)$ includes approximately 98% of the student grades and the lowest Program tier AMP Average predicted by Model 1 is 87.2% (tier 3), the majority of the students

will finish the Program with at least an 80% AMP Average. In fact, only 5 of the 171 students selected (from USMA AMP Class of 2011) earned an AMP Average < 80%.

The Student's Perspective

In order to look at Program selection from another perspective, from the eyes of the student, we administered an end of program survey. Students from USMA AMP Class of 2011 were given class time to complete 17 survey questions (5 multiple choice, 12 Likert-scale, and one open-ended) at the end of the 2007-2008 academic year (AY2008). The AMP Student Survey is shown in Appendix A; Table 6 and 7 show Survey results for Questions 1-16.

	Q1	Q3	Q5	Q8	Q10	Q11	Q12	Q13	Q14	Q15	Q16
Average Answer	1.9	2.0	1.7	2.7	3.4	3.2	2.9	3.3	2.5	2.8	2.0
Standard Deviation	1.1	1.1	1.0	0.7	1.0	1.1	1.2	1.2	1.1	1.1	1.0

Table 6: Likert-Scale Question Results for AMP Student Survey (USMA AMP Class of 2011).

	Q2	Q4	Q6	Q7	Q9
Answer (a)	26 (18%)	22 (16%)	70 (44%)	32 (21%)	37 (23%)
Answer (b)	49 (35%)	43 (31%)	33 (21%)	95 (62%)	53 (34%)
Answer (c)	27 (19%)	14 (10%)	14 (9%)	8 (5%)	49 (31%)
Answer (d)	37 (26%)	51 (36%)	3 (2%)	14 (9%)	14 (9%)
Answer (e)	2 (1%)	10 (7%)	38 (24%)	5 (3%)	5 (3%)

Table 7: Multiple Choice Question Results for AMP Student Survey (USMA AMP Class of 2011).

The results look promising – it seems students thought that they were placed within the correct mathematics track at the Academy. Survey Questions 1, 3, and 5 show that although the average AMP student felt slightly less confident about Program placement after completing the first course in the AMP, after finishing the second course confidence was up. This small dip in confidence is possibly due to the difference in high school verses college mathematics courses and the rigors of Academy life. This is supported by the results of Survey Question 4 – student *effort and study habits* and the *AMP curriculum* are the major factors influencing student confidence after completing the first course in the Program. Although the majority of the AMP students felt that their *effort and study habits* negatively affected their ability to perform at their maximum level throughout the Program, students thought that their *natural aptitude for mathematics* was the biggest factor for their success. These results strengthen the argument that we selected the most qualified students, previously defined as those who demonstrate the ability to learn at an increased rate. This is further supported by the results of Survey Questions 8 & 16 – the majority of

students agreed that, as a result of the AMP, they are more confident mathematical problem solvers and that the curriculum moved at the right pace and included the right amount of material.

In the Survey, students were also asked to answer questions pertaining to parts of the AMP curriculum not found in typical college calculus courses or sequences: the technical writing program, guest lecture series, and integration of technology. Here we received some results that were unexpected. The average AMP student neither disagrees nor agrees that the guest lecture series or the use of technology contributed to learning or increased interest in AMP course material. The technology related results are not surprising since student feedback throughout the Program have shown a strong dislike for the use of technology (due to a limited teaching of technology concepts in the classroom). However, we have always believed that the majority of students felt that the guest lecturers increased interest and learning in course material. The results related to the technical writing program are much more expected. Although students generally didn't feel that their involvement in group projects increased their interest in course material, the majority felt that projects increased learning in the AMP.

When students were asked, "*How many students should not have been selected for the Program in a typical section*?" the average reply was one student. This is about 11 students total (not including the approximately 25 students transferred out of the AMP either by self nomination or by the suggestion of their instructor in the first month of the Program). Although we were very interested in the students' response to this question, the results were for the most part expected. AMP Instructors, when asked the same question, agreed with the students. Instructors pointed to lack of student motivation and mathematical maturity as reasons, both of which are difficult to identify during the AMP selection process.

Identifying the Best AMP Students

In order to better look at the students that may not have fully benefited from the AMP (possible those discussed above) we looked at data from students that received an AMP Average of ≤ 83%, shown in Table 8.

Tier	# Students	AP-CALC Average Score	USMA VAL Average Score	SAT-M Average Score	AMP Average
1	1	-	79.8 %	720	80.8 %
2	2	5	68.0 %	715	79.6 %
3	8	4.3	50.3 %	685	81.3 %
4	4	3	70.8 %	693	79.8 %
5	7	-	55.4 %	647	79.7 %
All	22	4	58.6 %	679	80.4 %

Table 8: Performance Data for USMA AMP Class of 2011 Students with AMP Average ≤ 83%.

A further analysis shows that, as a benchmark, students that have a SAT-M of ≤ 680, a USMA VAL of ≤58%, and an AP-CALC score ≤ 4 should be considered high risk. However, we must be cautious since of the 22 total students (in the USMA AMP Class of 2011) that fall into this group only ten are included in Table 8. The additional 12 students, initially considered high risk, scored > 83% AMP Average. Identifying the students that are truly at risk during the selection process makes for a difficult task!

AMP Student Survey results from the group of 22 students included in Table 8 are shown below in Tables 9 and 10.

	Q1	Q3	Q5	Q8	Q10	Q11	Q12	Q13	Q14	Q15	Q16
Average Answer	2.6	3.3	2.6	2.1	3.3	3.1	3.3	3.7	2.6	3.0	2.3
Standard Deviation	0.9	1.0	0.9	0.7	1.1	1.1	1.0	1.0	0.7	0.8	1.0

Table 9: Likert-Scale Question Results for AMP Student Survey for USMA AMP Class of 2011 Students with AMP Average ≤ 83%.

	Q2	Q4	Q6	Q7	Q9
Answer (a)	8	7	3	9	2
Answer (b)	6	4	7	7	10
Answer (c)	3	2	4	1	7
Answer (d)	1	4	0	1	1
Answer (e)	0	2	5	1	0

Table 10: Multiple Choice Question Results for AMP Student Survey for USMA AMP Class of 2011 Students with AMP Average ≤ 83%.

The Survey results for this group paint a slightly different picture as compared to the USMA AMP Class of 2011 as a whole. In general, these students were not very confident that they were placed in the correct USMA core mathematics track. Also, students within this group questioned their natural aptitude for mathematics and felt that the AMP curriculum moved a bit fast and included too much material. Overall, students in this group don't seem to be ideal candidates for the AMP and as a result didn't fully benefit from the Program.

Discussion of Results and Conclusion

Over the past three years we have found the USMA VAL and tiered selection process a helpful and effective means of selecting students for the AMP. Although successful, this process does not account for all significant performance indicators and may be omitting important selection data that's readily available. The benefits for the use of the SAT-M score coupled with the USMA

VAL and a predictive model approach for student selection has the potential to make the selection process easier and more efficient. A sensible course of action may be to use Model 3 as an AMP selection tool when performance information for AMP candidates includes AP-CALC data; however, when AP-CALC information is not available, Model 1 can be used as an alternative. This may allow for a more efficient and effective selection process.

In general, the AMP Student Survey results are a nice verification that we are currently selecting the right students for the Program. Although there are parts of the Program that can be improved, such as the guest lecture series and technology integration, we believe that the majority of the students benefit greatly from the curriculum. As identified earlier, students that have a SAT-M of $\leq$ 680, a USMA VAL of $\leq$58%, and an AP-CALC score $\leq$ 4 should be considered high risk. A possible course of action is to interview students in this group before enrollment in the Program (and possibly during the first month of the Program for students selected) to better understand their motivations and maturity level.

Over the next year we will continue to examine the use of predictive models for student selection. We will carry out student selection for the 2009 academic year with the use of both the tiered and predictive model selection processes and compare the results. The ongoing statistical validations of the connections between student success and proper performance indicators will serve to legitimize the selection of students for the Advanced Mathematics Program at the U.S. Military Academy.

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References

Bridgeman, B. & Wendler, C. (1989). *Prediction of grades in college mathematics courses as a component of the validity of SAT-Mathematics scores.* (College Board Report No 89-9). New York, NY: College Entrance Examination Board.

Odell, P. & Schumacher, P. (1995). Placement in first year mathematics courses using scores on SAT-math and a test of algebra skills. *PRIMUS*, 5, 61-72.

Retchless, T., Boucher, R., & Outing, D. (2008). Calculus Placement that Really Works! M*AA Focus*, 28.1, 20-21.

Rueda, N. & Sokolowski, C. (2004). Mathematics Placement Test: Helping Students Succeed. *The Mathematics Educator*, 14.2, 27-33.