a. This question is quite simple once you envision the parallelogram as a rectangle instead. By doing this it is clear that the question becomes the following: How many unique paths exist in a 4 x 5 Cartesian coordinate system connecting two diagonally opposite points in which all paths are grid-like relative to the Cartesian system.

It is clear that for this specific system/ network, all paths must include 4 vertical subpaths and 3 vertical subpaths in order to connect corner to corner. Thus 7 subpaths must join 8 nodes (letters in this problem). From combinatorix, therefore, we know the number of unique paths, and thus ways to spell "ARMY MATH," is represented by

$$\frac{7!}{3!4!} \text{ or } \binom{7}{4}$$

Easily computed, there are 35 unique paths in which to spell ARMY MATH in the given configuration.

To expand this to any phrase consisting of two words with the same number of letters (*n*), we simply construct a generalized network of nodes. So in this general case, the network would consist of *n x n+1* nodes. Thus 2*n*-1 subpaths, *n* vertical and *n-1* horizontal, would be needed to complete the path from corner to corner. And just as before, the number of paths, by combinatorix, is represented by the expression

$$\frac{(2n-1)!}{n!(n-1)!} \text{ or } \binom{2n-1}{n}$$

Geometrically, the idea behind the latter combination is that in order to get from point A to point B, 2*n*-1 links subpaths must be made. However we must 'choose' *n* of those subpaths, no more no less, to be in a certain direction (based on an arbitrary configuration of the Cartesian network). Hence (2*n*-1) C *n* (which is equivalent to (2*n*-1) C (*n*-1), making the Cartesian configuration arbitrary).

b. The first battle is to determine what *N*(*i*) is. To do this, we break it into two parts: a) *i*=even, b) *i*=odd. Now we begin looking for patterns. We see that we have a choice between either a short jump (1 space) or a long jump (2 spaces). In order to get from 0 to 4 we must move a total 4 spaces via either method. The possibilities are:
   a. 4 short jumps (0 long jumps)
   b. 2 short jumps (1 longs jump)
      i. Short, short, long
      ii. Short, long, short
      iii. Long, short , short
   c. 0 short jumps (2 long jumps)

   Therefore, there are 5 unique ways to get from 0 to 4
   In order to get to 0 to 5 we must move a total of 5 spaces via either method. The possibilities are:
   a. 5 short jumps (0 long)
   b. 3 short jumps (1 long)
      a. Short, short, short, long
      b. Short, short, long, short
      c. Short, long, short, short

        d. Long, short, short, short
    c. 1 short jump (2 long)
        a. Short, long, long
        b. Long, short, long
        c. Long, long, short

Therefore there are 8 unique was to get from 0 to 5.

Now we must look at our path decision making a different way. Let's start with moving from 0 to 4.

a. Way A: 4 total jumps (none long), mathematically equiv to 4 C 0.
b. Way B: 3 total jumps (2 short, 1 long), mathematically equiv to 3 C 1.
c. Way C: 2 total jumps (2 long), mathematically equiv to 2 C 2.

Thus the total number of unique paths from 0 to 4 is equal to
4 C 0 + 3 C 1 + 2 C 2 = 8

Now 0 to 5:

a. Way A: 5 total jumps (none long), mathematically equiv to 5 C 0.
b. Way B: 4 total jumps (3 short, 1 long), mathematically equiv to 4 C 1.
c. Way C: 3 total jumps (1 short, 2 long), mathematically equiv to 3 C 2.

Thus the total number of unique paths from 0 to 5 is equal to
5 C 0 + 4 C 1 + 3 C 2 = 11

Therefore we can extrapolate this pattern to two different function for both cases (*i*=even or *i*=odd)

$$N(i = even) = \sum_{n=0}^{i/2} (i-n)Cn$$

$$N(i = odd) = \sum_{n=0}^{(i-1)/2} (i-n)Cn$$

In computing a couple terms it is quick to see that *N* is equivalent to *F* (where *F* is the Fibonacci sequence). The first terms of *N* are 1,2,3,5,8,13…

The next question therefore is quite simple. $\frac{N(1+i)}{N(i)}$ does converge because $\frac{F(1+i)}{F(i)}$ converges. Because we all are familiar with famous recurring mathematical expressions we know that the sequence converges to φ the "Golden Ratio" approximately 1.618.

And lastly *N*(2007) =
1984707997851919030111407603630214584217016555399348701476922
6250884574313938083350795882749332422950140847740016581724394
2324567298371808617274593168211714748509936647709836525836069
7851262940923954285413543361714752530158978859616700531 86

2079962439263844056648509049844527707648590618251589128148 37
9503826747397404328074864246255191853526100319462790593601 37
0743459999078494496877620011031142546453694387085011678977 71

Mathematica code:

f[j]=

$$\sum_{n=0}^{\frac{j-1}{2}} \frac{(j-n)!}{n!\,(j-2n)!};$$

f[2007]