

Problem 3: Keeping it \mathbb{R} eal

USMA D/Math Problem of the Week

Submission Deadline: October 4, 2007 at 1600

Circle one: cadet faculty non-usma student non-usma faculty other

Problem Statement: The polynomial $f(x) = x^3 - x^2 - x + 1$ intersects the x -axis precisely twice, at $x = -1$ and $x = 1$. It also has a local maximum at $x = -1/3$. In this case, the distance from $-1/3$ to 1 is twice that from -1 to $-1/3$.

(a) Show that this is not a coincidence: if a *cubic polynomial has a double real root*, so it has the form $g(x) = (x - r)^2(x - b)$, and if the local extremum between the roots occurs at $x = p$, then $|p - r| = 2|p - b|$.

(b) Find the polynomial of *smallest order* which has (i) all real roots, (ii) intersects the x -axis precisely twice at $x = 1$ and $x = 6$, and (iii) has a local maximum (or minimum) at $x = 3$.

Submit your answer to Dr. Elisha Peterson at ae3263@usma.edu as an attachment to your email, with the subject line **WP POTW**. Or drop your solution off in my mailbox or on my desk (with date and time please!)