Problem 12: Odd Numbers in Pascal's Triangle

USMA D/Math Problem of the Week

Submission Deadline: February 14, 2008 at 1600

Circle one:	cadet		faculty		non-usma student		non-usma faculty		other	

Problem Statement: Pascal's triangle is formed by letting each number in a row be the sum of the two numbers directly above it. The first eight rows of the triangle are shown below. If my calculations are correct, 75% of the numbers shown are odd.

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}
$$

What percentage of numbers in *all of* Pascal's triangle are odd? In particular, if p_n represents the percentage of numbers in the first n rows of Pascal's triangle that are odd, find the limit of p_n as n approaches ∞.

Submit your answer to Dr. Elisha Peterson at ae3263@usma.edu with the subject line **WP POTW**... or drop your solution off in my mailbox or on my desk (with date and time please!)