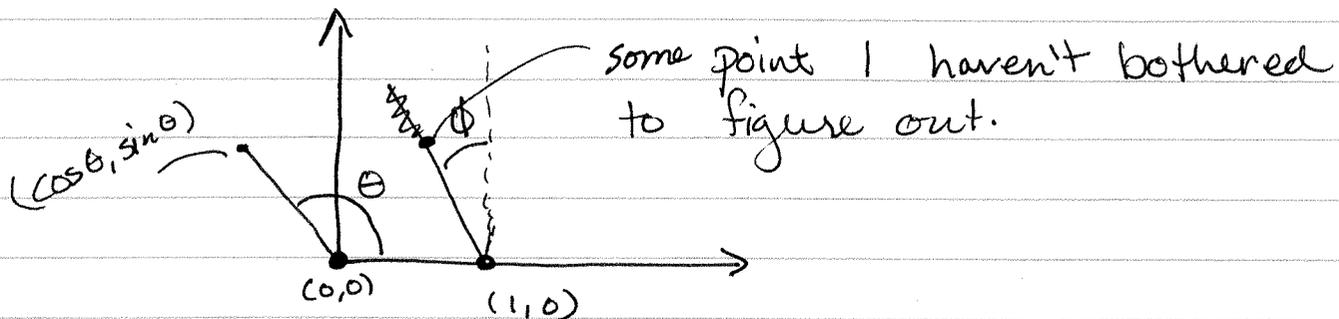


For simplicity, normalize the trees so they have height 1 unit and are ~~one~~ 1 unit apart.

The uniformity must be in terms of the angle at which the trees fall.

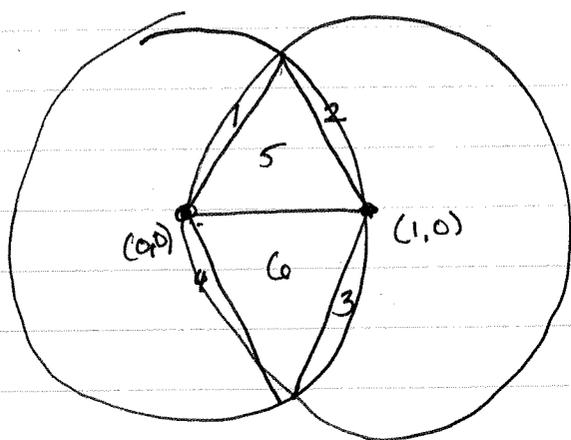
Suppose tree 1 is at $(0,0)$ and the angle of its fall is θ . Tree 2 is at $(1,0)$ and the angle of its fall is ϕ , as defined below:



To normalize the probability, we say

$$\int_0^{2\pi} \int_0^{2\pi} k d\phi d\theta = 1 \quad \text{so} \quad k = \frac{1}{4\pi^2}.$$

The following diagram indicates 6 regions where the falling trees may cross:



Region 5 corresponds to $0 \leq \theta \leq \pi/3$

and $\pi/6 \leq \phi \leq \pi/2$. If the trees both fall in this area, they will cross.

Region 6 is of the same size, so we only complete the probability of region 5, which

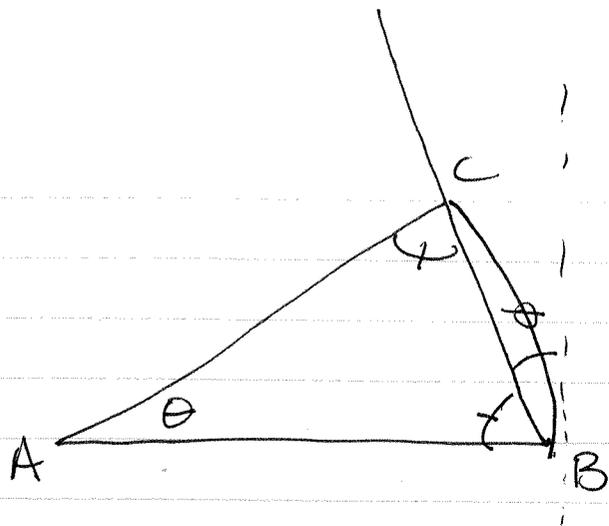
$$\frac{\sqrt{3}}{4} \text{ is } \int_0^{\pi/3} \int_{\pi/6}^{\pi/2} r^2 d\phi d\theta = \frac{\pi^2}{9} \cdot \frac{1}{4\pi^2} = \frac{1}{36}$$

Thus the probability that both trees fall in region 5 or both trees fall in region 6 is $2/36$.

Regions 1, 2, 3, and 4 are also symmetric,

so we examine only region 2. For the trees to cross, the angles must be right.

Fix the angle θ where the first tree falls. Since $\phi \geq \pi/6$ is in Region 5, we consider only $\phi < \pi/6$.



This diagram indicates the minimum angle ϕ^0 so that the trees cross. Specifically, it is $\theta/2$, because the triangle ABC is isosceles.

So the probability of a crossing in Region 2 is

$$\int_0^{\pi/3} \int_{\theta/2}^{\pi/6} k \, d\phi \, d\theta = \frac{1}{144}$$

The symmetry of the regions gives the total probability as

$$4\left(\frac{1}{144}\right) + 2\left(\frac{1}{36}\right) = \underline{\underline{\frac{1}{12}}}$$

ANS