CORE MATHEMATICS

DEPARTMENT OF MATHEMATICAL SCIENCES

U S M A

ACADEMIC YEAR 2005 – 2006

CORE MATHEMATICS AT USMA
AY 05-06

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INTRODUCTION

This document is designed to inform USMA faculty in the Math/Science/Engineering (MSE) Departments, and interested others, about the core mathematics program. Inside you will learn what mathematical skills and concepts you can expect from your cadets as they progress from admission to the end of the core math sequence. You will see some of the philosophy and growth goals that we have adopted in order to develop a diverse group of high school graduates into college juniors who are prepared to succeed in an engineering stem or in higher-level disciplinary study. You will read about our Interdisciplinary Lively Applications Projects (ILAP) and Liaison Professor programs. These programs aim to achieve a more integrated MSE experience for cadets by promoting coordination and collaboration between the Department of Mathematical Sciences and other academic departments. You will also learn the details of our program for identifying and reinforcing required mathematical skills for entering cadets.

We have included a detailed summary of course objectives for each core math course for this academic year. As part of our educational philosophy, we recognize that mastering conceptual knowledge is a difficult process requiring periodic review, practice, and consolidation. Therefore, we recommend that courses which rely heavily on portions of this conceptual material identify those portions to the cadet at the beginning of the course, and then reinforce student understanding as appropriate.

Also of special interest is the list of Mathematical Recall Knowledge. This is a modest list of basic facts that the MSE Committee has judged should be memorized by each cadet. Within the core math program, cadets periodically test their proficiency on this current and accumulated recall knowledge. For MSE courses that rely heavily on some subset of these recall skills, we again recommend that you identify these to your cadets at the beginning of the course, and then reinforce (and test) them as appropriate.

Applied mathematics is the process of appropriately transforming one form into a more useful form in order to reveal additional insight. This can be as simple as transforming an array of numbers into a sum; a function into its derivative; or a computer network into a bipartite graph. A historian who can transform the data of secondary sources into information that reveals the interactions of an earlier society is an applied mathematician in disguise! It is essential that we provide our cadets an experience where they can appreciate and apply the power of transformations to solve real problems.

Technologies are facilitating new transformations and have made other transformations obsolete. Hence, the teaching of mathematics at the undergraduate level is changing! While new technologies allow us to do innovative things today, experience strongly suggests that much of it will be only partly relevant, sometimes misleading, and occasionally wrong. We know that what we consider to be the fundamentals of mathematical knowledge change more slowly. Our role is to continually find the appropriate balance between technology and the fundamentals as mathematical pedagogy and content evolve.

We hope you find the information in this booklet useful. We welcome your feedback on how we can better coordinate our programs, and on what information we can include here in order to help you succeed as a teacher who uses the tools from core mathematics. The Department of Mathematical Sciences updates this document annually in coordination with the MSE Committee. Please direct any comments to the former for inclusion in the next edition.

GARY W. KRAHN
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EDUCATIONAL PHILOSOPHY

The Role of Core Mathematics Education at USMA

The mind is not a vessel to be filled but a flame to be kindled. -- Plutarch

Core mathematics education at USMA includes both acquiring a body of knowledge and developing thought processes judged fundamental to a cadet's understanding of basic ideas in mathematics, science, and engineering. Equally important, this educational process in mathematics affords opportunities for cadets to progress in their development as life-long learners who are able to formulate intelligent questions and research answers independently and interactively.

At the mechanical level, the core math program seeks to minimize memorization of a disjoint set of facts. The emphasis of the program is at the conceptual level, where the goal is for cadets to recognize relationships, similarities, and differences to help internalize the unifying framework of mathematical concepts. To enhance understanding of course objectives, major concepts are presented numerically, graphically, and symbolically. This helps cadets develop a visceral understanding that facilitates the use of these concepts in downstream science and engineering courses.

Concepts are applied to representative problems from science, engineering, and the social sciences. These applications develop cadet experience in modeling and provide immediate motivation for developing a sound mathematical foundation for future studies.

The core mathematics experience at USMA is not a terminal process wherein a requisite subset of mathematics knowledge is mastered. Rather, it is a vital step in an educational process that enables the cadet to acquire more sophisticated knowledge more independently. Cadet development dictates that we must provide the cadet time for experimentation, discovery, and reflection. Within this setting, review, practice, reinforcement, and consolidation of mathematical skills and concepts are necessary and appropriate, both within the core math program and in later science and engineering courses.

Cadets completing the core math program will have developed a degree of proficiency in several modes of thought and habits of the mind. Cadets learn to reason deductively, inductively, algorithmically, by analogy, and with the ability to capture abstractions in models. The cadet who successfully completes the USMA core mathematics program will have a firm grasp of the fundamental thought processes underlying discrete & continuous processes, linear & nonlinear dynamics, and deterministic & stochastic processes. The cadet will possess a curious and experimental disposition, as well as the scholarship to formulate intelligent questions, to seek appropriate references, and to independently and interactively research answers. Most importantly, cadets will understand the role of applied mathematics – insight gained from transformations.

Adaptive Curriculum

Computing has changed profoundly-and permanently-the practice of mathematics at every level of use. College mathematics departments, however, often lag behind other sciences in adapting their curricula to computing, although considerable momentum is now building within the community for greater use in computing.… Computing can enhance undergraduate study in many ways. It provides natural motivation for many students, and helps link the study of mathematics to study in other fields. It offers a tool with which mathematics influences the modern world and a means of putting mathematical ideas into action. It alters the priorities of courses, rendering certain favorite topics obsolete and making others, formerly inaccessible, now feasible and necessary. Computers facilitate earlier introduction of more sophisticated models, thus making instruction both more interesting and more realistic. The penetration of computing into undergraduate mathematics is probably the only force with sufficient power to overcome the rigidity of standardized textbooks. The power of technology serves also an epistemological function by forcing mathematicians to ask anew what it means to know mathematics. Those who explore the impact of technology on education indict introductory mathematics courses for imparting to students mostly skills that machines can do more accurately and more efficiently.

The department of mathematical sciences is committed to taking a lead in developing an effective new mathematics curriculum that attempts to foresee the mathematical needs of tomorrow's students. Our goal is to create a four-semester mathematics program that will enhance the mathematical maturity and problem solving skills of students.

Technology has reversed the roles of calculus and modeling. Traditionally, calculus determined the core program and modeling was used to support the application aspect of the program. Tomorrow, modeling and inquiry will determine the program with calculus and other mathematics subjects supporting the modeling portion of the program. Placing an emphasis on both discrete and continuous modeling broadens the role of mathematics to include transforming real world problems into mathematical constructs, performing analysis, and interpreting results. Placing an emphasis on inquiry provides opportunities for student growth in terms of learning how to learn, becoming an exploratory learner, and taking responsibility for one's own learning.

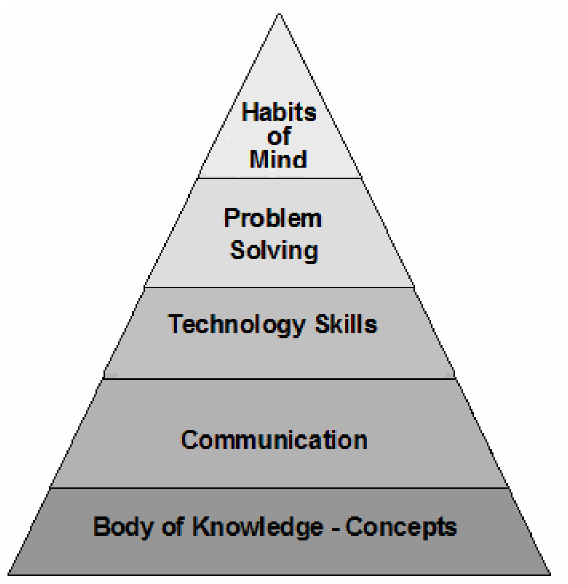
Our approach is to provide students with a broad appreciation and practice of mathematics through modern ideas and applications. The program is designed to provide student experiences through important historical problems and noteworthy contemporary issues as well. The goal is for students to establish a foundation from which to address unanticipated problems of the future. Examples of problem domains that will be considered include: the fair distribution of resources among nations; scheduling transportation resources; network design; financial models; population models; statistical inference; motion in space; optimization models; position, location, and geometric models; accumulation models; growth and change models; long-term behavior of systems; algorithm analysis, numerical techniques; linear and non-linear systems, and heuristic techniques. Some problem domains will be revisited several times at more sophisticated levels during the 4-semester program as students develop into more competent problem solvers.

The program will include contrasting approaches to problem solving such as: Continuous and Discrete; Linear and Non-linear; Deterministic and Stochastic; Deductive and Inductive; Exact and Approximate; Local and Global; Quantitative and Qualitative; Science (what is) and Engineering (what can be). For example, mathematics has the responsibility to develop a broad range of reasoning skills. Exposure to the art of reasoning will come through the process of induction and deduction. We will create learning opportunities where students move from the *puzzling* data to a suggested meaning (i.e., induction). Just as importantly, we will require students to move from the suggested meaning back to the data (i.e., deduction). This process of reasoning, along with other threads (e.g. approximation with error analysis, data analysis, …, discovery), will run throughout the course.

Central to the entire program is the concept of problem solving in the modeling sense. This involves: 1) understanding the problem; 2) devising a plan to solve the problem; 3) carrying out the plan; 4) looking back to examine the solution in the context of the original problem.

Core Mathematics Program Goals

Core Mathematics Program Goals: The goals of the Core math program support the USMA Academic Program goals and the Math-Science goals. The five primary goals of the core math program are provided below.



- **Acquire a Body of Knowledge:** Acquiring a body of knowledge is the foundation of the core math program. This body of knowledge includes the fundamental skills requisite to entry at USMA as well as the incorporation of new skills fundamental to the understanding of calculus and statistics.

- **Apply Technology:** Technology can change the way students learn. Along with increased visualization, computer power has opened up a new world of applications and solution techniques. Our students can solve meaningful real-world problems by leveraging computer power appropriately.

- **Communicate Effectively:** "Students learn mathematics only when they construct their own mathematical understanding". The successful problem solver must be able to clearly articulate their problem solving process to others.

- **Develop Habits of Mind:** Learning is an inherently inefficient process. Learning how to teach oneself is a skill that requires maturity, discipline, and perseverance. The core math program seeks to improve each cadets reasoning power by introducing multiple modes of thought. These modes of thought include deduction, induction, algorithms, approximation, implications, and others.

- **Build Competent and Confident Problem Solvers:** The ultimate goal of the core math program is the development of a competent and confident problem solver. Students need to apply mathematical reasoning and recognize relationships, similarities, and differences among mathematical concepts in order to solve problems.

STUDENT GROWTH GOALS

Transforming High School Graduates into College Juniors

The greatest good you can do for [students] is not just to share your riches but to reveal to [them their] own. -- Benjamin Disraeli

When cadets enter the core math sequence, they are only a few months past their high school graduation. They represent a variety of backgrounds, levels of preparation, and attitudes toward problem solving. At the end of the core math sequence, these same cadets will have chosen an upper-division engineering sequence and a major (or field of study), will be halfway through the requirements for a BS degree, and will be aggressively and successfully tackling the more synthetic and open-ended challenges of their engineering science and engineering design courses. One of the responsibilities of the core math program is to move this diverse group of entering cadets from the former state to the latter. Our vehicle for doing this is the set of student growth goals outlined below. We feel that success in the MSE program at the Academy requires that each cadet mature in attitude toward the nature of problem solving and mathematics. Each cadet must grow confident in learning and applying mathematics, and each must develop facility in the skills and arts that allow them to apply mathematics and to collaborate with others. Careful coordination of the four core math courses allows a cadet to grow in each of the following areas. The result is a cadet who is prepared to perform the synthesis and to meet the open-ended challenges required by their chosen engineering stem.

BODY OF KNOWLEDGE:

Incoming Students

- students are expected to be competent in Algebra, Trigonometry and Pre-calculus upon entry to USMA.
- although many students have been introduced to calculus prior to entering USMA, the curriculum assumes that students possess no prior knowledge of calculus.

MA103

- Fundamental skills in context to real-world problem solving
- Problem Solving
- Basic Matrix Algebra
- Discrete Models and Difference Equations
- Intro to Calculus

MA104

- Understand average and instantaneous rates of change
- Understand the concept of the derivative
- Be able to take **basic** derivatives by hand (polynomials, trig, exponential, and logarithmic functions)
- Understand vectors and motion in space
- Understand partial derivatives, directional derivatives, and gradients and use these concepts to solve problems.
- Model problems using differential equations

MA205

- Understand Accumulation in One and More Variables (Single and Multiple Variable Integration)
- Use integrals to model and analyze simple physical problems.
- Evaluate integrals and iterated integrals.
- Solve 1^{st} and 2^{nd} order ODEs analytically (separation of variables and the characteristic equation)
- Analyze 1^{st} and 2^{nd} order ODEs numerically, graphically, and analytically using technology.

MA206

- Understand the basic concepts of probability, statistics, and random variables.
- Model applied problems using the fundamental probability distributions.
- Make inferences about a population using Point Estimation, Interval Estimation, and Hypothesis
- Use Linear Regression to make predictions.

TECHNOLOGY SKILLS:

- We expect incoming students to be comfortable using basic computer technology.
- Throughout the core curriculum students will use both EXCEL and Mathematica to visualize, solve, analyze, and experiment with a myriad of mathematical functions (discrete, continuous, linear, non-linear, deterministic and stochastic).
- Following the core sequence, we expect students to be confident and aggressive problem-solvers that use technology to leverage there ability to solve complex problems.

COMMUNICATION

- Incoming students are introduced to technical writing in their first core math course and will do technical writing throughout the core program.
- Following the core math sequence, we expect students to adequately synthesize their thoughts to explain their problem solving processes both orally and in the form of a well written technical paper.

HABITS of the MIND:

- In studying mathematics cadets learn good scholarly habits for progressive intellectual development.
- Cadets learn that mathematics is an individual responsibility that requires the motivation to learn, effort, time, and interaction with others.
- Cadets learn that mathematics requires an experimental disposition that in turn requires a curious mind, the ability to recognize patterns, the ability to conjecture, and the ability to reason by analogy.

PROBLEM SOLVING

- The core math program is a mathematical modeling and problem solving based curriculum.
- The core curriculum begins by reviewing essential skills in a modeling and problem solving context.

At the conclusion of the core math sequence it is anticipated that students are confident in their abilities to attempt to solve problems which they may have or have not ever seen before. In essence, we would like our students to confidently and aggressively pursue a solution process, when they are not really sure what to do.

MA103 – Mathematical Modeling and Introduction to Calculus
AY 05-06 - Class of 2009
COURSE OBJECTIVES

1. Introduce the cadets to mathematical modeling and problem solving. This first course in the core mathematics sequence introduces problem solving techniques to formulate and structure mathematical models. The course utilizes the enhanced capabilities of technology to achieve numerical and graphical results. The majority of the course topics are introduced via multi-day real world experiences. These experiences illustrate the direct applicability to the many tools and problems solving techniques discovered in the course. Throughout the course, the relationship to calculus is investigated and discussed. The course is broken down into three primary blocks of instruction. The block objectives are as follows.

A. Problem Solving and Modeling with Functions
   (1) Overview of a problem solving process that focuses on:
      - Understanding the problem
      - Devising a plan
      - Carrying out the plan
      - Looking back
   (2) Familiarize cadets with the general shapes, characteristics and limitations of several common functions (power, exponential, logarithmic and trigonometric).
   (3) Determine the appropriate function and its parameters to best model a phenomenon, given a specified data set. Understand the limitations and capabilities of this model.
   (4) Interpret model results to answer questions or make predictions about the situation being investigated.

B. Modeling with Sequences and Calculus
   (1) Apply modeling process to problems involving change at discrete intervals (e.g. populations, decay, interest rates).
   (2) Model a situation by formulating difference equations. Solve the model numerically (through iteration), graphically, and analytically (for linear cases).
   (3) Analyze the equilibrium and the long-term behavior of a system of difference equations.

C. Modeling with Linear Systems and Markov Chains & Uncertainty
   (1) Model situations with a system of linear equations and solve using linear algebra techniques.
   (2) Familiarize cadets with several common linear algebra skills (transformations, matrix multiplication, inverses, and elementary row operations).
   (3) Through the use of Markov chains, model situations involving uncertainty.

2. Develop cadets' modeling abilities through in-class exercises and group projects. These events require cadets to analyze real-world problems, make assumptions, model the system, solve the model, and then interpret the results.

3. Develop cadets' abilities to employ technology as an analytical tool.

   A. Use technology to aid in the problem solving process. Adapt models to changes in variables, parameters and conditions.

   B. Use a Computer Algebra System (Mathematica) to graph functions; solve systems of linear equations; find determinants, examine long-term behavior, and iterate Markov Chains.

C. Use computer spreadsheets (EXCEL) to develop numerical and graphical models, their solutions and their long-term behavior.

4. Practice technical expression through student writing. Articulate and interpret technical results on projects in formatted technical reports. Illustrate written communication skills through essay questions and project write-ups. Illustrate verbal communication skills through project presentations and briefing in-class work.

MA104 - CALCULUS I
AY 05-06 - Class of 2009
COURSE OBJECTIVES

1. Expose cadets to problems in the continuous domain involving one or more independent variables, and introduce them to methods for analyzing and solving these types of problems in differential calculus:

A. Single Variable Differential Calculus.
   (1) Develop an understanding of limits, continuity and derivatives.
   (2) Analyze properties of functions using the 1^{st} & 2^{nd} derivative.
   (3) Model problems involving the optimization of functions of one variable.

B. Motion in Space.
   (1) Understand properties of vectors and differentiation of vector functions.
   (2) Parameterize curves and analyze parameterized functions.
   (3) Model/analyze problems incorporating vector functions and 3D space (intersection/collision/projectile motion/motion in 3D space).
   (4) Understand and be able to apply the concepts of dot product and cross product.

C. Multivariable Differential Calculus.
   (1) Develop an understanding of functions of more than one variable and their behavior.
   (2) Develop an understanding of partial derivatives, directional derivatives, and gradients.
   (3) Model problems involving the optimization of functions of more than one variable.

D. Ordinary Differential Equations (ODEs).
   (1) Understand what an ordinary differential equation is and what its solution represents.
   (2) Use graphical (slope and direction fields) and numerical (Euler's method) techniques for analyzing ODEs.
   (3) Model problems involving growth/decay, motion, spring-mass systems, heating/cooling and mixing.

2. Develop cadets' modeling abilities through classroom applications, homework, concept based exams and Modeling and Inquiry exams, as well as through one major project requiring 6-8 hours of work. The project requires making assumptions about and modeling a realistic situation, using appropriate technology, and writing a technical report. The technical report includes a summary of results, as well as appendices containing the process of problem solving and other analytical, graphical, numerical and verbal support.

3. Develop cadets' abilities to employ computers as analytical tools. Use a Computer Algebra System (Mathematica) to find limits, find roots, solve systems of equations, graph algebraic and elementary transcendental functions, symbolically differentiate, compute basic vector operations, evaluate total and partial derivatives, plot multivariable functions and space curves, approximate solutions to differential equations, plot slope fields of DE's, and complete graded Modeling and Inquiry exams and the project.

4. Develop cadets' written and verbal skills. Analyze, interpret, and articulate results of course projects (#2 above) in written technical reports. Present analysis and results of problems and projects verbally in class. Complete several writing assignments during the semester.

MA205 - CALCULUS II
AY 05-06 – Class of 2008
COURSE OBJECTIVES

1. Analyze problems in the continuous domain involving one or more independent variables.

   A. Motion in 3D Space (Vector Calculus)
      (1) Parameterize curves and analyze parameterized functions.
      (2) Understand vectors and be proficient in their use.
      (3) Become proficient in vector parameterization.
      (4) Apply single variable integral calculus to vector functions to model and analyze motion problems.

   B. Accumulation in One and More Variables (Single and Multiple Variable Integration)
      (1) Use integrals to model and analyze simple physical problems.
      (2) Evaluate integrals and iterated integrals.
      (3) Be able to convert points and equations between Cartesian and polar forms to evaluate double integrals).

   C. Ordinary Differential Equations (ODEs)
      (1) Model problems involving growth/decay, motion, heating/cooling, and mixing.
      (2) Be acquainted with graphical (slope fields) and numerical (Euler's method) techniques for analyzing ODEs.
      (3) Solve 1^{st} and 2^{nd} order ODEs analytically (separation of variables and the characteristic equation) and using a computer algebra system.
      (4) Be able to discuss long term behavior of the system (stable, unstable)j

   D. Mathematics Specific to Cadets' Majors and Engineering Stems: Cadets are sectioned by Major/FOS (MSE) or Engineering Stem (HPA) and taught from syllabi developed in conjunction with partner disciplines.

2. Develop cadets' modeling abilities through classroom activities, graded homework assignments, Modeling and Inquiry Problems and a project. The project will require making assumptions to model a realistic situation, using appropriate technology, writing a technical report and providing an oral presentation.

3. Develop cadets' abilities to employ computers (using Excel Spreadsheets and Mathematica) as analytical aids.

4. Practice communication of complex, technical concepts through student writing and oral reports.

MA206 - PROBABILITY & STATISTICS
AY 05-06 - Class of 2008
COURSE OBJECTIVES

1. Gain an understanding of how to model uncertainty and solve problems with those models.

A. Data Analysis. Gain an ability to analyze and model univariate and bivariate data sets.

(1) Understand the relationships between a population and a sample.
(2) Describe a data set (location, central tendency, symmetry/skewness).
(3) Create the empirical distribution function (EDF) for a univariate random sample.
(4) Model the EDF with a parametric model by minimizing the sum of squared error (SSE).
(5) Create a scatterplot with paired data from a random sample.
(6) Model the paired data with a linear parametric model, minimizing SSE.
(7) Assess the fit of a model with a sum of squares metric.

B. Random Variables. Understand that Random Variables are models of real-world, uncertain phenomena. Also, understand certain mathematical operations that enable random variable models to solve problems. Specifically understand the underpinnings of Monte Carlo simulation and use it to solve probabilistic problems.

(1) Define and classify random variables (discrete, continuous, and joint).
(2) Derive, understand and use PDFs and CDFs (uniform, binomial, gamma, exponential, normal, t, $\chi 2$).
(3) Understand that the CDF models the EDF.
(4) Use computer algebra systems to implement the uses of PDFs and CDFs.
(5) Find expected values and variances of random variables.
(6) Apply knowledge of binomial, gamma, uniform, and exponential random variables to Monte Carlo Simulation.
(7) Understand how to conduct a simple method of moments estimation for model parameters.
(8) Understand and apply the Central Limit Theorem. Understand that the CLT is the major building block for the statistical inference part of the course.

C. Understand how to transform sample data into inference about population parameters.

(1) Understand and apply the basic concepts of point estimation.
(2) Under appropriate conditions, calculate and interpret confidence intervals for the mean and/or variance.
(3) Set up and perform hypothesis tests; understand how to calculate and interpret level of significance with P-values.
(4) Understand and perform simple linear regression modeling, and interpret regression output. Understand the probabilistic nature of the regression coefficients. Be able to predict with confidence intervals, using the regression model.

2. Develop cadets' modeling abilities through classroom applications (#1 above) and through group and/or individual projects. Each project typically requires student interpretation of a realistic situation, requires the cadet to identify any simplifying assumptions, asks open-ended questions, requires model refinement, and requires a written technical report.

3. Develop cadets' abilities to employ technology as analytical aids in solving problems. Emphasis is placed on determining the appropriate calculation to perform, understanding why the calculation is performed, and interpreting the results of the calculation.

4. Improve cadets' abilities to transform raw data into information for decision makers. Further develop their skills at explaining technical material, both verbally and in writing.

MA364 - ENGINEERING MATHEMATICS
AY 05-06 - Classes of 2006 and 2007
COURSE OBJECTIVES

1. Equip cadets with some of the basic mathematical tools used in their engineering studies, develop their expertise in the applied mathematics process, and make cadets comfortable with technology, e.g. *Mathematica* – a computer system for doing mathematics.

A. Fourier Series and Fourier Transforms
(1) Cadets develop criteria of least squares.
(2) Apply Fourier series to model various periodic functions over intervals.
(3) Analyze spectrum for a signal and discuss its meaning.

B. Ordinary Differential Equations - Single and Systems
(1) Model with first and second order differential equations, spring-mass-dashpot, RLC circuits, population (modest amount), etc.
(2) Model with systems of differential equations, e.g. tank mixing problems, kidney dialysis machine functioning.
(3) Convert second order ODE to linear system.
(4) Analyze ODE solution in terms of system response (phase angle and amplitude).
(5) Solve systems of ODEs by hand, by Laplace transforms, and by eigenvalues.
(5) Use solutions and eigenvalue information obtained from ODEs to predict behavior.

C. Laplace Transforms
(1) Understand the transformational approach of the Laplace transform solution strategy for differential equations.
(2) Present modest by hand work and turn to technology for transforming.
(3) Solve initial value problems with Laplace transforms.
(4) Introduce rationale for Laplace and special functions Unit Step and Dirac in modeling context and interpret results.
(5) Interpret the Laplace Transform, i.e. see the Transfer Function and the System Function leading to the system Response.

D. Accumulation/integration
(1) Present enormous mixture of problems USING accumulation as success strategy.
(2) Set up problems with little “element of stuff” – no matter what the application.
(3) Always use technology (*Mathematica*) for evaluating integrals.
(4) Check for reasonableness of answers.

E. Vector Calculus
(1) Understand the utility and applications of vector fields and of vector differential and integral operations.
(2) Visualize vector fields and operations and describe in terms of physical examples.

F. Introduction to Partial Differential Equations
(1) Understand the derivation of the heat/diffusion equation.
(3) Identify the utility of Fourier series representations in the presented closed form solution to the heat/diffusion equation; complete analytic solution is NOT covered.
(3) Model heat flow in a rod and through the ground using the diffusion equation.
(4) Build numerical methods using EXCEL spreadsheet to approximate the solution of the heat equation.
(5) Use *Mathematica*’s `NDSolve` command to solve PDEs and spend great deal of time examining plots and playing “what if” games with initial and boundary conditions.

G. Matrix Algebra
   (1) Solve systems of equations. In the Civil Engineering approach in fall set up Truss equations using the method of joints and solved as a complete system of equations.
   (2) Find and interpret eigenvalues and eigenvectors.
   (3) Understand the significance of the dominant eigenvalue and its corresponding eigenvector in the context of population models and other examples.

H. Support physics and engineering principles throughout, for example.
   (1) Free Body Diagram
   (2) Newton's Second Law of Motion – Change in Momentum equals sum of forces.

2. Develop cadets' modeling abilities through regular (daily!) graded homework and projects. Typically, there is one common project and one individually (or teams of two) selected and designed project.

3. Permit cadet discovery wherever possible – a small example occurs in developing least squares criteria for Fourier Series; a large example is in individually designed projects with teams of two. Cadets design their own investigation and pursue these ideas with several IPRs, a final write-up, and a formal presentation.

4. Develop cadets' abilities to use computers as tools in the problem solving process to obtain and verify solutions, as well as to present and communicate understandable results. *Mathematica* is used for algebraic manipulation, solving differential equations, doing Laplace transforms, evaluating integrals, determining eigenvalues and eigenvectors, operating on vectors, graphing results, and writing up results for submission with full documentation.

5. Improve cadets' ability to effectively communicate the results of their work. The reports submitted for each project are articulate and well organized. Summary comments and supporting work are expected to be arranged to present clearly to the reader the purpose of the project, procedure used, results obtained, conclusions drawn, and recommendations made.

6. Develop exceptional documentation skills and a sense that it is good to work with others and seek help when stuck on regular daily graded homework.

7. Typically, the fall semester addresses the mathematical needs of cadets majoring in Civil Engineering while the spring term addresses the needs of Mechanical, Nuclear, and Electrical Engineering majors.

INTERDISCIPLINARY LIVELY APPLICATIONS PROJECTS (ILAPs)
AY 05-06

The Department of Mathematical Sciences believes that cadets learn mathematics by doing mathematics. Traditionally, this has meant daily work that enables cadets to practice and explain their problem-solving skills, which enables instructors to gauge the cadet's understanding. While this practice continues, the advent of calculators, computers, and associated software now provides opportunities for cadets and teachers to create and explore more sophisticated problems. In developing these "more sophisticated problems" mathematics instructors here, as elsewhere, have naturally turned to mathematical applications in nature, science, engineering, economics, as well as to other fields including physical education and psychology. As a result, the mathematics faculty is often engaged in conversations dealing with how to best prepare cadets to wield these new weapons of discovery and learning. We formalized this process in our Interdisciplinary Lively Applications Project (ILAP) program.

ILAPs have been a part of the core mathematics program since AY 92-93. The goals of the ILAP program are to

- excite cadets with the power of mathematics as a tool in describing, investigating, and solving applied problems.
- excite cadets about the fields that become accessible to them as they master more mathematics.
- acquaint downstream departments with their future customers, allow these departments to demonstrate the utility of mathematics within their discipline, and involve these departments in exciting these cadets at an early stage of their development.
- enhance inter-departmental cooperation and make the four-year cadet experience more cohesive.

The ILAP program operates under the supervision of the Academy's Math, Science, and Engineering (MSE) Committee as part of a National Science Foundation project involving a consortium of 10 schools. Each year, the MSE Committee designates downstream ("application") departments and particular mathematics core courses to coordinate project development. Faculty members from the two departments formulate a suitable project in an applied area of interest in which a particular mathematical concept being studied in the core course is used.

Usually, each core math course presents from one to three projects during the course of the semester that demonstrate realistic applications of the mathematics being studied. In some cases, when the project is assigned to the class, the application department presents an introductory lecture (or video) explaining the project. When the project has been completed, the application department returns to present a concluding lecture (or video) reviewing the formulation and problem solving process, highlighting the important concepts being used, and illuminating extensions of the problem in their discipline. ILAPs that have been used in the core mathematics courses include the following:

MA103:

1-D Heat Transfer in a Bar – D/CME (Thermo) – 931
Pollution Levels in Lake Shasta – D/GEnE – 931
Tank Battle Direct Fire Simulation – D/SE – 931
Pollution Levels in the Great Lakes – D/GEnE – 941/942
Smog in the LA Basin – D/Chem – 941
Car Financing – D/SocSci (Econ) – 942
Pollution Levels in the Great Lakes – D/GEnE – 951
Car Financing – D/SocSci (Econ) – 951
Making Water in Space – D/Chem – 971
Brusselator – D/Chem – 981
Bioaccumulation of methylmercury – D/Chem – 001
Civil War –D/History – 0201

MA104:

Aircraft Ranges under Various Flight Strategies – D/CME (Aero) – 932
From Bungee Cords to the Trebuchet – D/CME (Vibes) – 932
Oxygen Consumption and Lactic Acid Production – DPE – 942
From Wing Resonance to Basketball Rims – D/CME (Vibes) – 942
Patient Scheduling & Profit Management – D/SocSci (Econ) – 952
Car Suspension System – D/C&ME (Vibes) – 952
Fighting Forest Fires – D/GEnE – 951/952
Flow Rates of A Reservoir Over a Dam – D/CME – 951/952
Parachute Jumping – D/Physics – 951/952
Modeling Engineer Operations in a Nation Building Scenario – D/CME – 971/972
Cut/Fill and Bridge Abutment/Span Computation – D/C&ME – 971/972
Railway Headwall Design – D/CME – 971/972
Water Contamination and Treatment – D/GEnE – 981/982
Forest Fire / Water Tank Properties – D/CME – 991/992
Earthquake Analysis of Water Tank – D/CME – 991/992
Dissolved Oxygen – D/GEnE – 011/012
Determining Speed and Distance of a Motorcycle Jump – D/EECS -- 022

MA205:

Vehicular Collisions – D/Physics – 951
Aircraft Utilization – D/SE – 951
Airborne Parachutists – D/Physics – 952
Patient Scheduling & Profit Management – D/SocSci (Econ) – 952
Tank Production – D/SocSci (Econ) – 961
Parachute Panic – D/Physics – 961
Math, the National Pastime? (Baseball) – D/Physics – 971
Cobb–Douglas Problem – D/SocSci (Econ) – 971
LAPES (Drop Zone) Problem – D/Physics – 971
Shuttle Problem – D/Physics – 981
Satellite Problem – D/CME – 981
Vector analysis of truss members – D/CME – 991
Modeling Bangalore Production (DDS /SV &MV Calculus) – D/SocSci (Econ) – 991
MTA Metrorail Connections – D/Soc Sci (Econ) – 992
Storage Tank Optimization – D/GEnE – 992
Flight of a TOW Missile – D/Physics – 001
Least Squares Regression of Data – D/Physics – 001
Counterbattery Target Acquisition – D/Physics – 011
Modernizing Camera Product Lines – D/SocSci (Econ) – 011
Incoming Interception in BMD – D/Physics – 021
Treating Motor Pool Runoff – D/GEnE – 021

MA206:

Pollution Levels in the Great Lakes – D/GEnE – 951/952
Statistical Analysis of Hudson River Pollution Data – D/GEnE – 981
Markov Chain Model of Dow Jones Industrial Average – D/SocSci– 982
Statistical Analysis of Elementary Circuits – D/Physics – 982
Call for Artillery – D/EECS – 981
The Dam Problem – D/CME – 992

The ILAP program embodies the belief that early mastery of mathematics skills produces in cadets the realization that they can indeed formulate and analyze interesting problems arising in engineering, science, business, and many other fields. Our experience is that this realization increases cadet motivation. We and the application departments have seen benefits of this increased motivation as cadets move from core mathematics into their disciplines.

Our ILAP material is public domain – any reader interested in these materials can find many of the ILAPs at the Department of Mathematical Science's web page (www.dean.usma.edu/math/), or can contact the ILAP coordinators, Prof. Don Small or LTC Archie Wilmer, for more information.

LIAISON PROFESSORS

"Servicing What We Sell"

Most departments at USMA rely on at least some portion of the core mathematics program to prepare cadets for studies in their Department. Additionally, the Department of Mathematical Sciences has found that the study of mathematical principles and methods is almost always made easier by considering problems in applied settings with realistic scenarios. In order to facilitate success in both of these areas, the Department of Mathematical Sciences has instituted a liaison program in which a tenured faculty member is designated as the Liaison Professor primarily responsible for coordination with a particular client Department. The designations for this academic year are as follows:

DEPARTMENT	LIAISON PROFESSOR	PHONE	OFFICE
BS&L (Human Factors)	LTC Andy Glen	x5988	TH237
Chemistry	LTC Jack Picciuto	x5608	TH225
C&ME	COL Joe Myers	x5611	TH250
EE&CS	COL Joe Myers	x5611	TH250
GEnE	LTC Rod Sturdivant	x3322	TH223
Physics	Fr. Gabe Costa	x2036	TH235A
	LTC Barbra Melendez	x7436	TH219
Soc Sci (Econ)	LTC Jerry Kobylski	x4014	TH227A
Systems Eng	LTC Andy Glen	x5988	TH237

COL Joe Myers has primary responsibility for oversight of the Liaison Professor program.

The major focus of this program is to achieve a more integrated student MSE experience by promoting coordination and collaboration between the Department of Mathematical Sciences and the other MSE Departments. The role of the Liaison Professor is to serve as the principal point of contact for members of the client department. The Liaison Professor fields questions, accepts suggestions, and works issues from the client department with respect to course material, ILAPs, procedures, timing, and any other matters of mutual interest. Additionally, the Liaison Professor is a first point of contact for course directors in the Department of Mathematical Sciences who are looking for examples and applications appropriate to their course and need referrals.

This program is not intended to preclude closer coordination (for instance, at the course director or instructor level), but rather is intended to provide a continuing source of information and input at the senior faculty level who can ensure that inter-departmental cooperation is accomplished across several courses in a consistent fashion, as required.

UNITED STATES MILITARY ACADEMY
MATHEMATICAL RECALL KNOWLEDGE

The following constitutes a basic mathematical vocabulary that will be built upon during each cadet's four-semester core mathematics experience and in his or her future math/science/engineering courses. Once each of these basic ideas has been covered in class, each cadet can be required to reproduce, upon demand in any future lesson of any math/science/engineering course, that idea exactly as shown here. Annotated beside each heading or item are the course number and lesson number that denotes the exact point in time when the cadet is responsible for each item (e.g.: 103/1 means that each cadet is responsible for knowing that item the first lesson of MA103). These items are recall knowledge - cadets are also required to be proficient in the more conceptual, less-verbatim ideas and skills reflected in each core math course's Course Objectives section of this document.

ALGEBRA (103/1)

1. $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. $a^b \cdot a^c = a^{b+c}$
3. $(a^b)^c = a^{bc}$
4. $\frac{a^b}{a^c} = a^{b-c}$
5. $y = \log_b x \Rightarrow x = b^y$
6. $\log_b b^x = b^{\log_b x} = x$
7. $\log_b x^a = a \log_b x$
8. $\log_b ac = \log_b a + \log_b c$
9. $\log_b \frac{a}{c} = \log_b a - \log_b c$
10. $\log_b a = \frac{\log_c a}{\log_c b}$

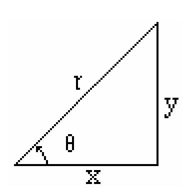
ANALYTIC GEOMETRY (103/1)

Rectangle:	Area = lw	Perimeter = $2l + 2w$
Circle:	Area = πr^2	Circumference = $2\pi r$
Rectangular Solid:	Volume = lwh	Surface Area = $2lw + 2lh + 2hw$
Cylinder:	Volume = $\pi r^2 l$	Surface Area = $2\pi r^2 + 2\pi rl$
Sphere:	Volume = $\frac{4}{3}\pi r^3$	Surface Area = $4\pi r^2$

Distance between (x_1, y_1) and (x_2, y_2) = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

TRIGONOMETRY (103/1)

With reference to the right triangle:



2π radians = 360 degrees

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $x^2 + y^2 = r^2$ $\sin^2\theta + \cos^2\theta = 1$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$

RELATIONSHIPS (MA103/1)

Corresponding sides of similar triangles are proportional

Distance = average rate $\times$ time

DIFFERENTIATION (MA 104)

1. $\dfrac{d}{dx}(a) = 0$ (104/11)
2. $\dfrac{d}{dx}(x) = 1$ (104/11)
3. $\dfrac{d}{dx}(au) = a\dfrac{du}{dx}$ (104/11)
4. $\dfrac{d}{dx}(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$ (104/11)
5. $\dfrac{d}{dx}(\mathrm{uv}) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ Product Rule (104/12)
6. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ Quotient Rule (104/12)
7. $\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$ Power Rule (104/11)
8. $\dfrac{d}{dx}[f(u)] = \dfrac{d}{du}[f(u)]\dfrac{du}{dx}$ Chain Rule (104/16)
9. $\dfrac{d}{dx}(\sin u) = \cos u\dfrac{du}{dx}$ (104/14)
10. $\dfrac{d}{dx}(\cos u) = -\sin u\dfrac{du}{dx}$ (104/14)
11. $\dfrac{d}{dx}(e^u) = e^u\dfrac{du}{dx}$ (104/14)
12. $\dfrac{d}{dx}(\ln \mathrm{u}) = \dfrac{1}{u}\dfrac{du}{dx}$ (104/14)

INTEGRATION (MA 205)

13. $\int a\,dx = ax + C$ (205/7)
14. $\int (u+v)\,dx = \int u\,dx + \int v\,dx$ (205/7)
15. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ (205/7)
16. $\int e^{ax}\,dx = \dfrac{e^{ax}}{a} + C$ (205/7)
17. Understand and be able to apply the Substitution Rule (205/10)
18. $\int \dfrac{du}{u} = \ln|u| + C$ (205/7)
19. $\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + C$ (205/7)
20. $\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + C$ (205/7)
21. If f is integrable on $[a, b]$, then $\displaystyle\int_a^b f(x)dx = F(b) - F(a)$ where $\dfrac{dF}{dx} = f(x)$

(Fundamental Theorem of Calculus) (205/8)

VECTOR CALCULUS (MA104)

22. $|\vec{A}| = \sqrt{a_i^2 + a_j^2 + a_k^2}$ (104/33)

23. $\vec{A} \bullet \vec{B} = a_i b_i + a_j b_j + a_k b_k = |\vec{A}||\vec{B}| \cos\theta$ (104/31)

24. $|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|\,|\sin\theta|$ (205/8)

25. $\nabla f = \frac{\partial f}{\partial x}\,\hat{i} + \frac{\partial f}{\partial y}\,\hat{j} + \frac{\partial f}{\partial z}\,\hat{k}$ (104/44)

PROBABILITY & STATISTICS (MA 206)

26. PDFs and CDFs: (206/16)

Discrete: PDF: $P(X = x) = p(x)$.

CDF: $P(X \le x) = \sum_{y \le x} p(y)$.

Continuous: PDF: $f(x)$ is used to find probabilities - $P(a \le X \le b) = \int_a^b f(x)dx$.

CDF: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy$.

27. The total accumulation of a probability distribution function is 1. (206/10)

Discrete: $\sum_{\forall x} p(x) = 1$ Continuous: $\int_{-\infty}^{\infty} f(x)dx = 1$

28. Calculate and interpret the expected value (mean) of a random variable. (206/11)

Discrete: $E(X) = \sum_{\forall x} x \cdot p(x)$ Continuous: $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$

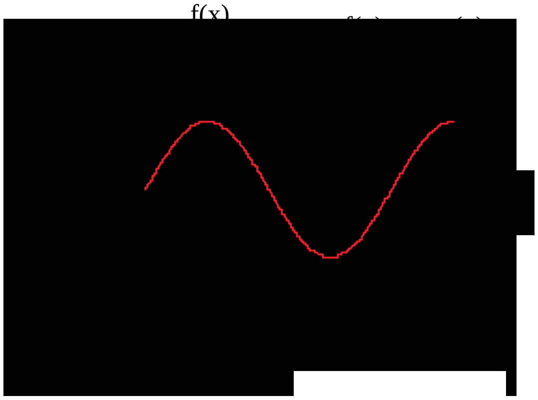
29. Calculate and interpret the variance of a random variable. (206/11)

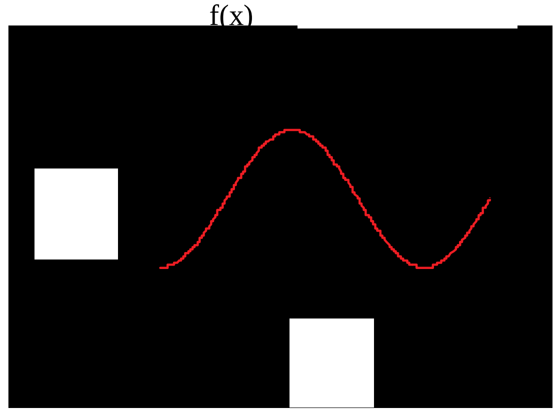
The variance is an expected value: $V(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$.

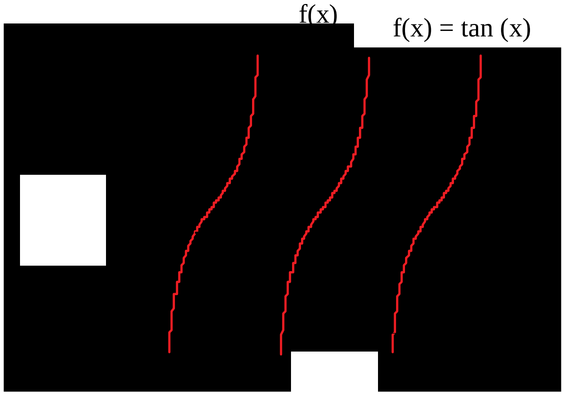
30. Percentiles of random variables including the median (50^{th} percentile). (206/17)

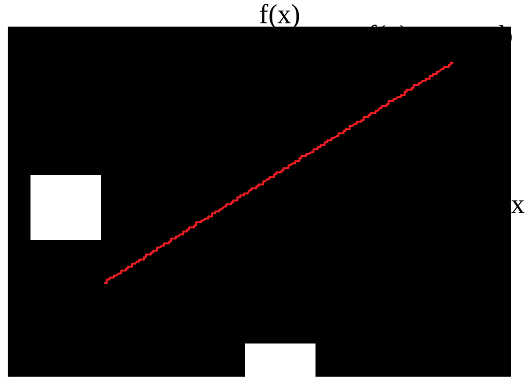
31. Central Limit Theorem: Let $X_1, X_2, \ldots, X_n$ be a random sample from *any* population with a finite mean and variance. Then, if *n* is sufficiently large, $\overline{X} = \sum X_i / n$ and $T_0 = \sum X_i$ are **approximately** normally distributed. (206/24)

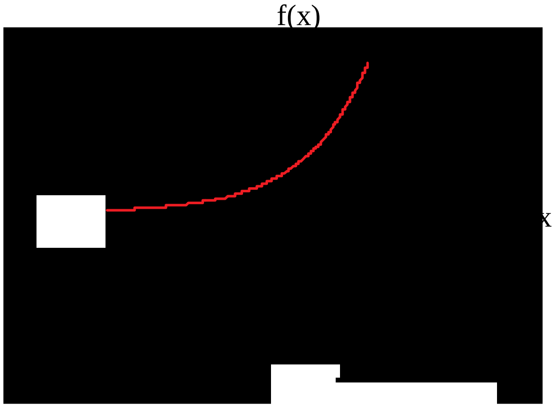
PROPERTIES OF FUNCTIONS (103/1)

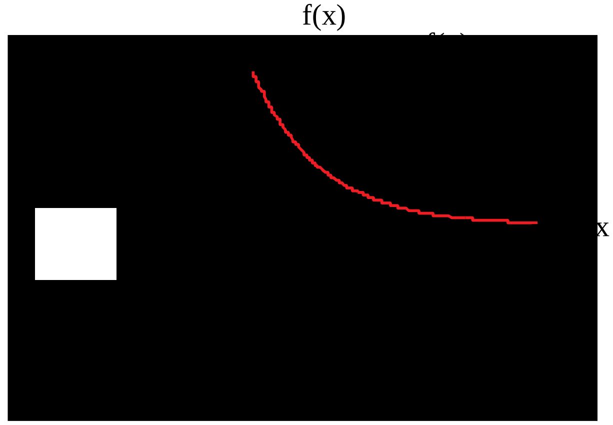


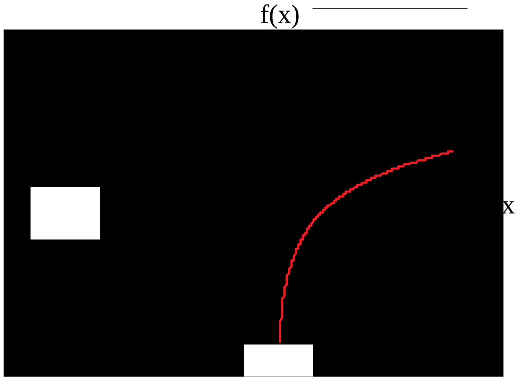












UNITED STATES MILITARY ACADEMY
REQUIRED SKILLS IN SCIENTIFIC COMPUTING

The following list constitutes the basic, required scientific computing skills that you will learn and use during your four-semester core mathematics experience and in your future math/science/engineering courses. Once each of the listed topics is covered in class, you must be able to execute this skill upon demand in any future lesson or course. Reference books, for the respective computing system, are the chief source of information for learning these skills. Annotated beside each item are the course and lesson numbers, which denotes the exact point in time you are responsible for that particular item (e.g. 103/1 indicates responsibility for that skill following the first lesson of MA103).

LAPTOP SKILLS (MATHEMATICA and MS EXCEL)

The following are all laptop (or desktop) computer skills. Most can be accomplished in either Mathematica or Excel, but some are easier in one package. As you become proficient in each skill, it is your job to decide which tool is the right one for the task at hand.

1. Enter a sequence model and evaluate analytically, numerically and graphically:

   E.g. Given the difference equation $\boldsymbol{a}_{\boldsymbol{n}+1} = 3\boldsymbol{a}_{\boldsymbol{n}} + \boldsymbol{n}^2$ (103/32)

   a. Find $\boldsymbol{a}_{100}$.

   b. Display a table of "n" and "$\boldsymbol{a}_{\boldsymbol{n}}$" values.

   c. Graph the time-series plot of $\boldsymbol{a}_{\boldsymbol{n}}$.

2. Perform basic matrix and vector operations
   a. Given 2 matrices, compute (if defined) their sum and product. (103/45)
   b. Find the transpose a given matrix. (103/45)
   c. Find the inverse of a given square matrix (if it exists). (103/46)
   d. Find the magnitude of a given vector. (104/29)
   e. Given 2 vectors, compute their dot product and cross product. (104/31-32)

3. Perform matrix operations on linear systems of equations in two and three variables:

   E.g. Given the linear system of equations, $\begin{bmatrix} 2 & 5 & 6 \\ 7 & 8 & 4 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$

   a. Solve the system of equations. (103/49)
   b. Perform row reduction operations on the augmented matrix. (103/49)

4. Find the roots of a polynomial or transcendental function (both real and complex):

   E.g. Find the roots of the functions (103/16)
   a. $r^3 - 7r^2 - 14r + 8 = 0$
   b. $e^x - 2 = 0$

5. Graph algebraic and elementary transcendental functions:

   E.g. Graph the following functions individually and together on the same axes. (103/17)
   $y_1 = 2x^2 + 3x - 24\ e^{2x}$
   $y_2 = \sin(3x) + e^x - 2\ln(x)$

6. Graph 3-dimensional objects (surfaces, space curves, and level curves):

E.g. Graph the space curve (104/26)
$\vec{r}(t) = t^2\,\vec{i} + 2t\,\vec{j} + \sin(t)\,\vec{k}\,,\ t \in [0, \pi]$

E.g. Graph the surface $z = \frac{x^2}{4} + \frac{y^2}{16}$. (104/34)

E.g. Graph the level curves for $x^2 + y^2 = k$ (104/34)

7. Evaluate limits: (104/5)

E.g. Evaluate $\lim\limits_{x \to 5} \frac{x-5}{x^2 - 25}$.

E.g. Evaluate $\lim\limits_{x \to \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7}$.

8. Evaluate derivatives:

E.g. Given the function $y = \frac{1}{x} + \sin(2x^3\text{-}x)$,

a. Find the derivative. (104/10)
b. Plot the derivative. (104/10)

E.g. Given the function $f(x, y) = \frac{1}{y} + \sin(2x^3 - y)$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. (104/41)

E.g. Given the function $f(x, y) = x^2 y + xy^2$,

a. Find the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial x^2}$. (104/41)

b. Find the gradient of $f(x,y)$ at the point (2,3). (104/44)

c. Find the directional derivative of $f(x,y)$ at the point (2,3) in the direction of $2\,\vec{i} - 4\,\vec{j}$. (104/43)

9. Solve nonlinear systems of equations:
a. Solving for relative extrema (104/45)
b. Constrained optimization (Lagrange) (104/47)

10. Evaluate definite and indefinite integrals:

E.g. Evaluate $\int_a^x f(y)dy$. (205/12)

E.g. Evaluate $\int (x\ln(x) - e^x)dx$ (205/12)

E.g. Evaluate $\int_0^{\pi} \left(x^2 + \cos(2x)\right)dx$. (205/7)

E.g. Evaluate $\int_0^3 \int_0^2 (x^2 y) dy\ dx$ (205/19)

E.g. Evaluate $\int_0^1 \int_0^{y^2} (3y^3 e^{xy}) dx\ dy$. (205/19)

E.g. Evaluate $\int_0^3 \sin\left[\frac{(2n-1)\pi x}{6}\right] \sin\left[\frac{(2m-1)\pi x}{6}\right] dx$, (364/33)

for $m = n$ and $m \neq n$.

11. Differential Equations (DE):

E.g. Given the DE $\frac{dy}{dx} = \cos(\pi x) + e^{-x} - 2$,

a. Plot the direction (slope) field of the DE. (104/1-8, 205/34)

b. Graphically approximate a solution curve to the DE for $y(0) = 2$. (104/1-8, 205/34)

c. Find a numerical approximation for $y(3)$ given $y(0) = 2$. (104/1-8, 205/35)

d. Verify a given solution to the DE (104/1-8, 205/36)

12. Probability and Statistics:

a. Fit a regression line to given data. (205/4, 206/35)

b. Compute the sample and population mean, median, variance, and standard deviation: (206/1)

E.g. Given the following data: 3.0 2.5 3.7 2.7 3.3, compute the mean, median, variance, and standard deviation.

c. Create an empirical CDF from a given matrix of data. (206/6)

d. Analyze data with descriptive statistics and descriptive plots. (206/3)

e. Simulate data (generate pseudo-random data) (206/13)

f. Perform linear regression and interpret the output (206/34)

13. Seek on-line HELP to learn new commands, uses, and methods, as well as to troubleshoot one's own efforts.

MS-WORD, with Graphics Objects from other Packages

14. Create a formal document in electronic form that includes textual, graphical, numerical, and analytical modeling and analysis. (103/17)

REQUIRED MATHEMATICAL SKILLS FOR ENTERING CADETS

Cadets enter the United States Military Academy (USMA) with different backgrounds in mathematics. To successfully begin work in math/science/engineering (MSE) courses at USMA, cadets must arrive with knowledge of certain mathematical skills and concepts. The MSE Committee deems the items below to be fundamental skills and concepts that all entering cadets must possess. Early in Cadet Basic Training, the Department of Mathematical Sciences administers a "Gateway" exam to assess fundamental skills. Therefore, we encourage incoming cadets to evaluate their mathematics skills and work at remediation of any deficiencies prior to arriving at USMA.

The Department of Mathematical Sciences administers two additional "Gateway" exams during the first semester in MA103 (Discrete Dynamical Systems and Introduction to Calculus). Self-paced texts and software files offer opportunities for self-remediation. Failure to pass at least one "Gateway" exam at mastery level (≥ 80%) may result in placement in MA100 (Precalculus) during the second semester instead of continuing into MA104 (Calculus and Differential Equations), or other scheduled remediation. Cadets enrolled in MA100 who do not demonstrate proficiency in these fundamental mathematics skills by the end of their first year may be separated from USMA.

Almost any high school algebra book is a suitable reference for these fundamental skills. An incoming cadet can obtain a DOS-based disk tutorial (3.5" PC format) by writing to "Department of Mathematical Sciences, USMA, West Point, NY 10996," or by calling (845) 938-4603/5673.

Note: All calculations must be done <u>without</u> the use of technology (i.e., calculator). Some examples of skills are provided in parentheses.

1. Algebra and Real Numbers.

a. Use symbols and operators to represent ideas and objects and the relationships existing between them.

b. Understand the relationship between measures of the physical world. (Velocity, distance and time: On a 40-mile car trip to Middletown, NY, you drive the first twenty miles at 40 mph and the last twenty miles at 60 mph. What is your average speed during the trip?)

c. Know and apply the following algebraic properties of the real number system: identity, associative, commutative, inverse, and distributive.

d. Express numbers using scientific notation. (Express 0.004312 in scientific notation.)

2. Radicals and Exponents.

a. Convert between radical and rational exponent form. (Transform $\frac{1}{\sqrt{x+2}}$ to the rational exponent form $(x+2)^{-\frac{1}{2}}$.)

b. Manipulate algebraic expressions that contain integer and rational exponents. (Simplify $4^{-\frac{3}{2}} \cdot 27^{-\frac{2}{3}}$.)

3. Algebraic Expressions.

a. Add, subtract, multiply, and divide algebraic expressions. (Find the remainder when $x^3 - 7x^2 + 9x$ is divided by $x - 2$.)

b. Simplify algebraic expressions. (Expand and simplify $(x-3)(x-2)(x-1)$.)

4. Factoring / Prime Numbers

a. Write a number as the product of factors. (Write 42 as the product of prime factors.)

b. Solve for the roots of a polynomial by factoring. (Find the roots of $x^2 - 5x + 6 = 0$.)

5. Linear Equations, Inequalities and Absolute Values.

a. Solve 2 simultaneous linear equations by graphing and by substitution. (Use a graph to estimate the point of intersection of the lines $2x + 3y = 7$ and $-x + y = 4$. Verify your result using back substitution.)

b. Solve linear equations and inequalities [graphically and algebraically]. (Solve $5(3-x) > 2(3-2x)$ for x.)

c. Solve linear equations and inequalities with absolute values. (Solve $|x - 4| \geq 3$ for x.)

6. Polynomials and Rational Inequalities.

a. Solve simple polynomial inequalities.
(Solve $x^2 + 3x + 6 > x - 4$ for x.)

b. Solve simple rational inequalities.
(Solve $\frac{x - 3}{x + 1} < 2$ for x.)

7. Straight Lines.

a. Determine the equation of a line. (Find the equation of a straight line passing through the points (2, 1) and (5, 4).)

b. Determine the equation of a line that is parallel or perpendicular to a given line. (Find the equation of a line parallel to the line $2y - 3x = 7$ and passing through the point (1, 2).)

8. Functions.

a. Identify the independent and dependent variables of a function.

b. Determine the domain and range of a real valued function. (Find the domain and range of the real valued function $g(x) = \frac{1}{x^2 - 2}$.)

c. Evaluate a function at a point. (Given $f(x) = 3x^2 - 2x + 4$, find $f(2a)$.)

d. Evaluate composite functions. (Given $h(r) = 3r^2$ and $g(s) = 2s$, find $h(a+2) - g(2a)$.)

9. Quadratic Equations and Systems

a. Solve for real and complex roots using the quadratic formula. (Find the roots of $3x^2 + 2x = -1$.)

b. Solve a system of quadratic equations in 2 variables by substitution. (Solve the system $y= 3-x^2$ and $y = 4 + 2x^2 - 2x$.)

10. Trigonometric Functions.

a. Define each of the 6 trigonometric functions (sin(θ), cos(θ), tan(θ), cot(θ), sec(θ), csc(θ)) in terms of the sides of a right triangle. ($\cos(\theta) = x/r$ where x is the adjacent side and r is the hypotenuse.)

b. Define each of the 6 trigonometric functions in terms of sin(θ) and cos(θ). ($\tan(\theta) = \frac{\sin \theta}{\cos \theta}$.)

c. Know the domain and ranges for the sine, cosine, and tangent functions.

d. Convert angle measures between degrees and radians. (Write 120 degrees as a radian measure.)

e. Memorize and use the 30/60/90 and 45/45/90 degree reference triangles.

f. Know and apply the trigonometric identity $\sin^2(\theta) + \cos^2(\theta)=1$. (Simplify the expression $2\cos^2(\theta) + \sin^2(\theta) - 1$.)

11. Logarithmic and Exponential Functions.

a. Know the relationship between logarithm and exponential functions [$y = \log_a x$, $a>0$, $a\neq 1$, is the inverse of the function $y = a^x$; $\log_a x=y \Leftrightarrow a^y=x$)]
(Evaluate $\log_3 27$.)

b. Know the properties of the logarithmic and exponential functions and use them to simplify logarithmic expressions. (Express as a single logarithm: $.5 \log_{10} x - \log_{10} y$.)

c. Solve simple logarithmic and exponential equations. (Solve the equation $3^{x+4} = 4$. for x.)

12. Graphs and Graphing.

a. Graph equations and inequalities. (Sketch a graph of the function $f(x) =3x^2 - 2x+7$ for $1<x<5$.)

b. Properly label a graph (axes, intercepts, asymptotes, and roots).

c. Know the general characteristics and shapes of the graphs of polynomial, logarithm, exponential and trigonometric functions.

d. Transform the graph of a known function. (From the graph of $f(x)$, graph the function $g(x) = 2f(x) - 3$.)

13. Analytic Geometry.

a. Know and apply the distance formula between 2 points. (Find the distance between the two points A(1,2) and B(-5,-3).)

b. Know and apply the circumference and area formulas for circles, triangles, and rectangles. If you double the radius of a circle, what happens to its circumference?)

c. Know and apply the surface and volume formulas for cylinders, spheres and rectangular solids.

d. Know the relationship between similar triangles. (A rectangle with base x and height 5 is inscribed in an isosceles triangle with base 10 and height 20. Determine x.)

e. Know and apply the Pythagorean Theorem to simple geometric problems. (Given a rectangle that is 4 ft by 7 ft determine the length of the diagonal.)

HISTORY OF THE DEPT OF MATHEMATICAL SCIENCES AT USMA

A teacher affects eternity; he can never tell where his influence stops. **--** Henry Adams

The Department of Mathematical Sciences, USMA, has a rich history of contributing to the education of cadets as confident problem solvers and of developing its faculty as effective teachers, leaders, and researchers. The story of mathematical education at West Point is full of interest: faculty curriculum developments, teaching methods and tools, and technological equipment. Many of the Department's advances have been exported outside the Academy to be employed by other civilian and military educational institutions.

EARLY BEGINNINGS: The actual teaching of mathematics at West Point dates from even before the Academy was established. In 1801, George Baron taught a few Cadets of Artillery and Engineers some of the fundamentals and applications of algebra. The Academy at West Point was instituted by act of Congress and signed into law by President Thomas Jefferson on 16 March 1802. The first acting Professors of Mathematics were Captains Jared Mansfield and William Barron. They taught the first few cadets topics in algebra, geometry, and surveying.

CONTRIBUTIONS TO THE NATION: Since the Academy was the first scientific and technical school in America, the early mathematics professors at USMA had the opportunity to make significant contributions not only to the Academy, but also to other American colleges. Perhaps the most prominent contributors were the early 19th century department heads Charles Davies and Albert E. Church. The work of these two professors had a significant impact on elementary schools, high schools, and colleges across the country. Davies became the Professor of Mathematics in 1823. He was one of the most prolific textbook authors of his day, writing over 30 books from elementary arithmetic to advanced college mathematics. His books were used in schools throughout the country from grade school to college. He had tremendous influence on the educational system of America throughout the 19th century. Albert Church succeeded Davies in 1837, and served as Department Head for the next 41 years. Another influential author, he published seven college mathematics textbooks.

PRODUCING LEADERS FOR THE NATION: Faculty from the Department have been notable military leaders for the country. Robert E. Lee was a standout cadet-instructor in the Department, Omar Bradley served as an Instructor for four years, Harris Jones and William Bessell were Deans of the Academic Board at USMA for a total of 15 years, and Department Heads Harris Jones, William Bessell, Charles Nicholas, John Dick, and Jack Pollin served impressively during two world wars.

The unique technical curriculum in place at the Academy during the middle of the 19th century produced many successful mathematicians and scientists for the country at large. West Point graduates Horace Webster, Edward Courtenay, Alexander Bache, James Clark, Francis Smith, Richard Smith, Henry Lockwood, Henry Eustis, Alexander Stewart, and William Peck filled positions as professors of mathematics or college presidents at other schools such as the U. S. Naval Academy, Geneva College, University of Virginia, University of Pennsylvania, University of Mississippi, Yale, Brown, Harvard, Columbia, Virginia Military Institute, Cooper Institute, and Brooklyn Polytechnic Institute. Two mathematics department heads became college presidents after leaving USMA; Alden Partridge founded and became the first president of Norwich University, and David Douglass served as president of Kenyon College in Ohio for four years. Jared Mansfield was appointed surveyor-general of the Northwest Territory, and Ferdinand Hassler became superintendent of the United States Coastal Survey. Capable individuals such as these exported the West Point model of undergraduate mathematics education throughout the nation.

While the faculty at USMA has been primarily military, the Department has benefited from civilian visiting professors since 1976. As part of the goal for civilianization of 25% of the faculty by 2002, begun in 1992, the Department established in 1994 a Center for Faculty Development in Mathematics. This Center establishes faculty development models and curricula and provides for the development of the "Davies Fellows", who serve as rotating civilian faculty members.

HISTORICAL AND LIBRARY HOLDINGS: Sylvanus Thayer's first task before assuming the Superintendancy in 1817 was to tour the technical institutions of Europe and assess what features USMA could use to advantage. One of Thayer's many accomplishments was to obtain numerous mathematics and science books from Europe. Thayer's book collection included many of the finest books available at that time. His books provided a solid foundation for the USMA library to build upon. Today, the West Point Library has one of the finest collections of pre-20th century mathematics books in the world. Also during the middle of the 19th century, the Academy instructors used elaborate physical models made by Theodore Oliver to explain the structures and concepts of geometry. This magnificent collection of string models is still in the Department today.

CURRICULAR DEVELOPMENT: After Thayer studied the military and educational systems of Europe, he reorganized the Academy according to the French system of the Ecole Polytechnic. The Department of Mathematics faculty (which included as Professor the distinguished scientist and surveyor Andrew Ellicott, and the famous French mathematician Claude Crozet whom Thayer recruited during his European trip to bring to USMA and America his expertise in Descriptive Geometry, advanced mathematics, and fortifications engineering) combined the French theoretical mathematics program with the practical methods of the English to establish a new model for America's program of undergraduate mathematics. This program of instruction in Mathematics grew over several decades and was emulated by many other schools in the country. The initial purpose of the Military Academy was to educate and train military engineers. Sylvanus Thayer, the "Father of the Military Academy" and Superintendent from 1817-1833, instituted a four-year curriculum with supporting pedagogy to fulfill this purpose. Thayer's curriculum was very heavy in mathematics; from Thayer's time to the late 1800's, cadets took the equivalent of 54 credit hours of mathematics courses. The topics covered in these courses were algebra, trigonometry, geometry, descriptive geometry (engineering drawing), analytic geometry, and calculus. Over the years, the entering cadets became better prepared and fewer of the elementary subjects were needed. During Davies' tenure (1823-37), calculus was introduced as a requirement for all cadets, and was used in the development of science and engineering courses. The time allotted for the mathematics curriculum decreased to 48 credit hours by 1940, and to 30 credit hours by 1950. During the 1940's, courses in probability & statistics and in differential equations were introduced into the core curriculum and a limited electives program was started for advanced students. In the 1960's, department head Charles Nicholas (previously one of the organizers of the Central Intelligence Agency) wrote a rigorous and comprehensive mathematics textbook (the "Green Death") that cadets used during their entire core mathematics program. With this text, he was able to adapt the mathematics program to keep up with the increasing demands of modern science and engineering. In the 1970's, Academy-wide curricular changes provided opportunities for cadets to major in mathematics.

During the 1980's, a mathematical sciences consulting element was established that allowed faculty members and cadets to support the research needs of the Army. This type of research activity continues today in the Army Research Laboratory (ARL) Mathematical Sciences Center of Excellence and in the Operations Research Center (ORCEN). In 1990 the Department introduced a new core mathematics curriculum that included a course in discrete dynamical systems, with embedded linear algebra. In that same year, the department changed its name to the Department of Mathematical Sciences to reflect broader interests in applied mathematics, operations research, and computation.

TECHNOLOGICAL DEVELOPMENT: USMA has a long history of technological innovation in the classroom. It was Crozet and other professors at USMA in the 1820's who were the first professors in the nation to use the blackboard as the primary tool of instruction. In 1944, the slide rule was issued to all cadets and was used in all plebe mathematics classes. During William Bessell's tenure (1947-1959), the mathematics classrooms in Thayer Hall (a converted riding stable) were modernized with overhead projectors and mechanical computers. Bessell was also instrumental in establishing a computer center at West Point. The hand held calculator was issued to all cadets beginning in 1975, and pre-configured computers were issued to all cadets beginning in 1986. In recent years, the department has established a UNIX workstation lab, an NSF-funded PC lab, and has run experimental sections with notebook computers and with multimedia.

NATIONAL COMPETITION: In the spring of 1933 West Point entered an interesting competition in mathematics. After USMA defeated Harvard at a football game the previous fall, a chance remark from President Lowell of Harvard to Superintendent Jones of USMA led the two schools to arrange a mathematics challenge match between the two schools with the two competing teams each composed of 12 second-year cadets. Army was the home team, so the Harvard competitors traveled by train to West Point. All the competitors took a test written by the president of the Mathematics Association of America. The **New York Times**, which had promoted the event with a series of articles in its sports section, reported the results. The West Point "mathletes" defeated Harvard in the competition that was the precursor to the national Putnam Competition. Since its inception in 1984, the Academy has entered two three-person teams in the International Mathematics Competition in Modeling. USMA won the top prize in 1988, 1993, 2000, 2001, 2002, 2003, and 2004.

During the first half of the 1990's, the Department of Mathematical Sciences at USMA became recognized as one of the more progressive mathematics programs in the country. The Department developed a strong "7 into 4" program that is exciting as well as innovative. Throughout the 1990's and into the 21st century, the Department of Mathematical Sciences has led the nation in curricular reform and in nurturing interdisciplinary cooperation and collaboration between and among partner disciplines. Project INTERMATH, a National Science Foundation grant to develop integrated curriculum and ILAPs, assisted in supporting these efforts. The USMA mathematics program has had great influence on mathematics education in America throughout its history, and strives to continue contributing to the improvement of mathematical education in America.

For a more extensive exposition on the history of the Department, refer to http://www.dean.usma.edu/math/about/history/dept.htm.