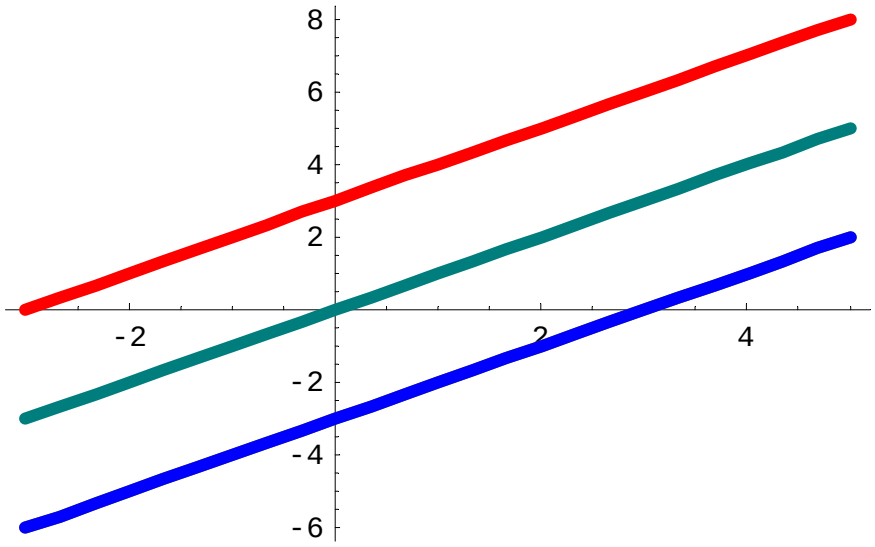
Functions & Lines Solution

Vertical and Horizontal Shifts (Stewart text, Section 1.3, page 38):

Suppose $c>0$. To obtain the graph of

1. $y = f(x)+c$, shift the graph of $y = f(x)$ a distance of c units upward.
2. $y = f(x)-c$, shift the graph of $y = f(x)$ a distance of c units downward.
3. $y = f(x-c)$, shift the graph of $y = f(x)$ a distance of c units to the right.
4. $y = f(x+c)$, shift the graph of $y = f(x)$ a distance of c units to the left.

1. Graph the function $f(x)=x$.



Write the resulting equations and graph those equations that are obtained from the graph of $f(x)$ with the following modifications:

a. Shift 3 units upward: $y = f(x) + 3 = x + 3$

b. Shift 3 units downward: $y = f(x) - 3 = x - 3$

c. Shift 3 units to the right: $y = f(x - c) = x - 3$

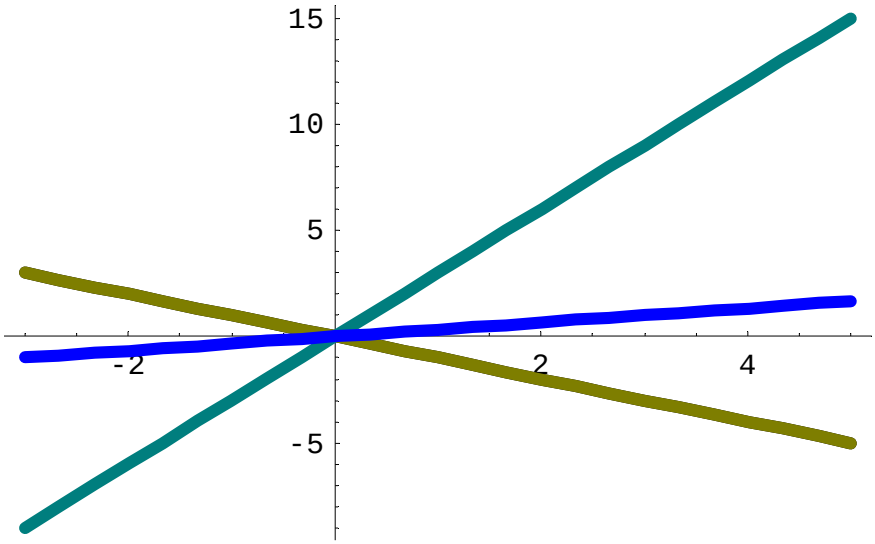
d. Shift 3 units to the left: $y = f(x + c) = x + 3$

Vertical and Horizontal Stretching and Reflecting (Stewart text, Section 1.3, page 39):

Suppose $c>1$. To obtain the graph of

1. $y = c*f(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c.
2. $y =(1/ c)*f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c.
3. $y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c.
4. $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c.
5. $y = -f(x)$, reflect the graph of $y = f(x)$ about the *x-axis*.
5. $y = f(-x)$, reflect the graph of $y = f(x)$ about the *y-axis*.

2. Graph the function $f(x)=x$ again.



Write the resulting equations and graph those equations that are obtained from the graph of $f(x)$ with the following modifications:

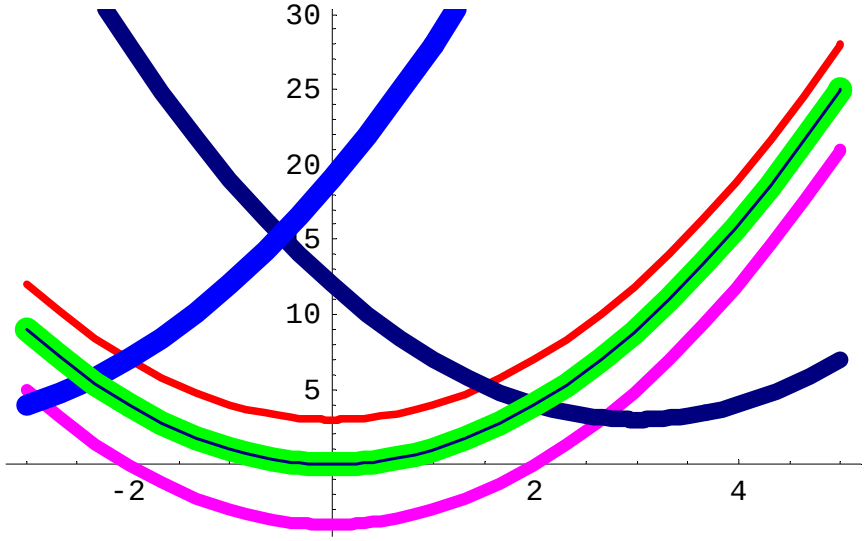
a. Reflect about the x-axis: $y = - f(x) = - x$

b. Reflect about the y-axis: $y = f (-x) = - x$

c. Stretch vertically by a factor of 3: $y = f (3 x) = 3x$

d. Shrink vertically by a factor of 3: $y = f (x/3) = x/3$

3. Graph the function $f(x)=x^2$. *(in black)*



On the same axes above, graph the following modifications using transformations:

a. $y=x^2+3$ *(in red)*

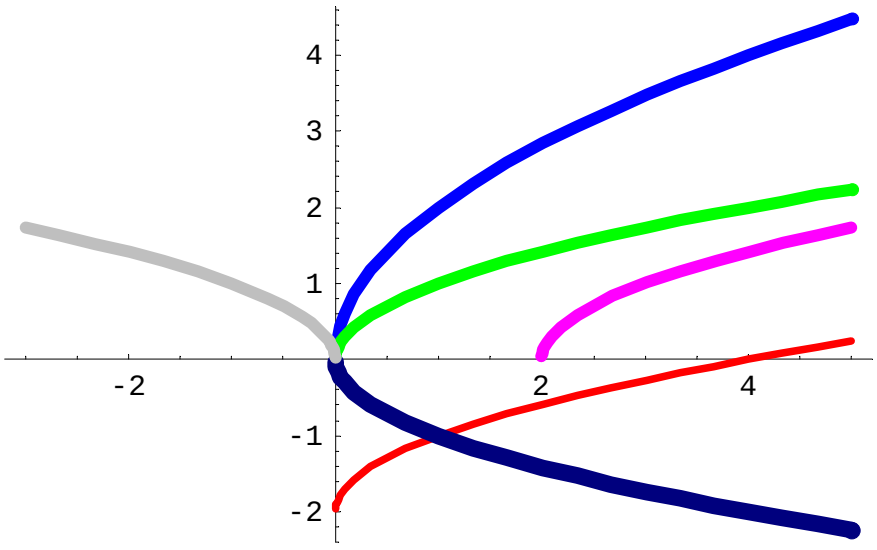
b. $y=x^2-4$ *(in purple)*

c. $y=(x-3)^2+3$ *(in dark blue)*

d. $y=(x+4)^2+3$ *(in light blue)*

e. $y=|x^2-1|$ *(in green)*

4. Graph the function $f(x)=\sqrt{x}$. *(in green)*



On the same axes above, graph the following modifications using transformations:

a. $y = \sqrt{x} - 2$ *(in red)*

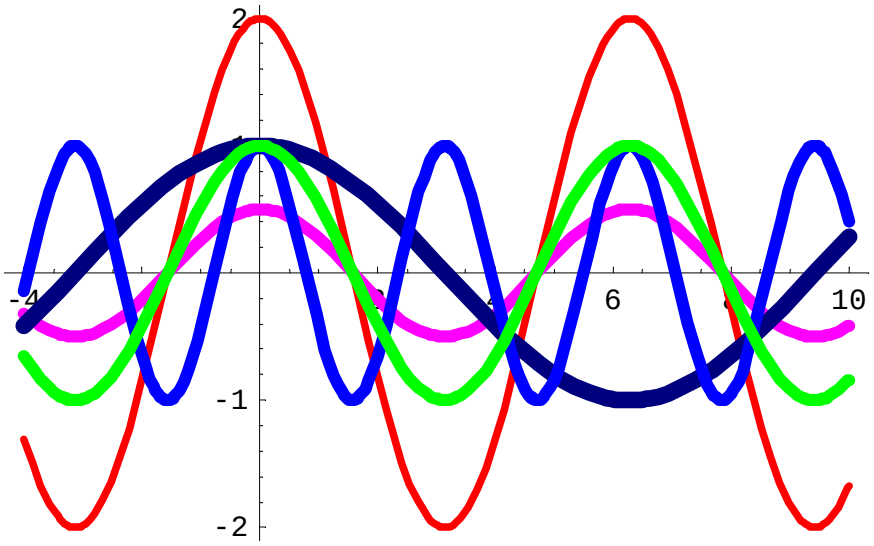
b. $y = \sqrt{x - 2}$ *(in purple)*

c. $y = -\sqrt{x}$ *(in black)*

d. $y = 2\sqrt{x}$ *(in blue)*

e. $y = \sqrt{-x}$ *(in gray)*

5. Graph the function *f(x)=cos (x). (in green)*



On the same axes above, graph the following modifications using transformations:

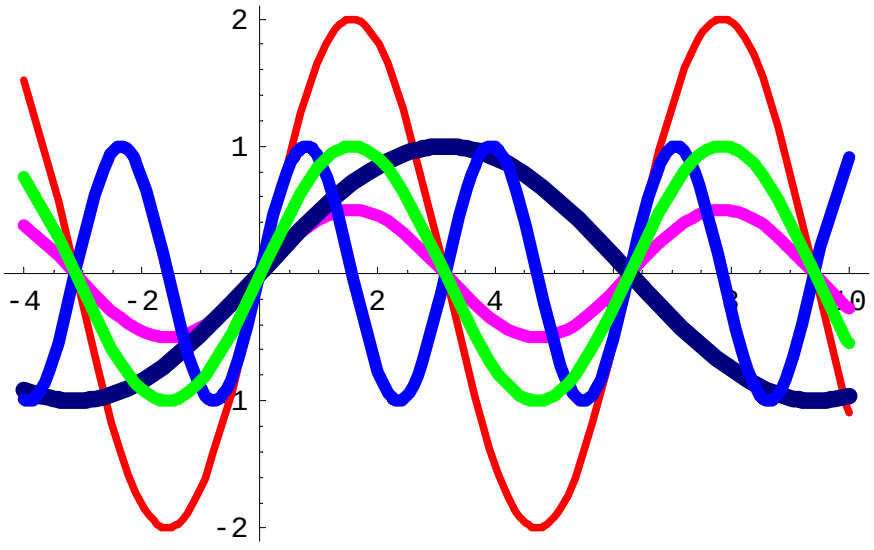
a. *y=2 cos (x) (in red)*

b. *y=(1/2) cos (x) (in purple)*

c. *y=cos(x/2) (in dark blue)*

d. *y=cos (2x) (in blue)*

6. Graph the function *f(x)=sin (x). (in green)*



On the same axes above, graph the following modifications using transformations:

a. *y=2 sin (x) (in red)*

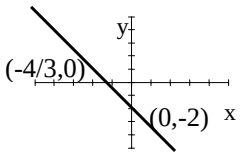
b. *y=(1/2) sin (x) (in purple)*

c. *y=sin(x/2) (in dark blue)*

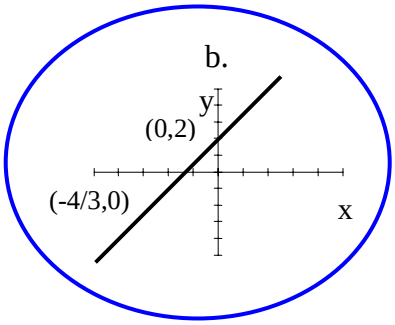
d. *y=sin (2x) (in blue)*

7. The graph of the function 2y – 3x = 4 is:

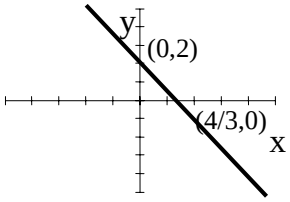
a.



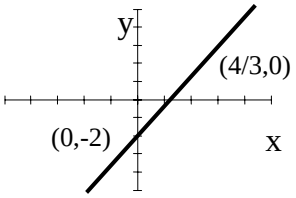
b.



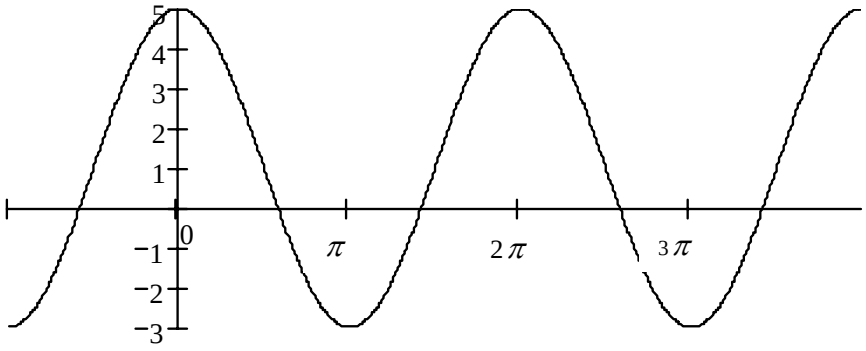
c.



d.



8. What is the equation of the following graph?



a. 4 sin(x) + 1

b. 4 cos(x) + 1

c. 5 sin(x)

d. 5 cos(x)

9. Given $f(x) = \sin x$ and $g(x) = 1 - \sqrt{x}$, find the functions $f \circ g, g \circ f, f \circ f, g \circ g,$ and state their domains.

f[g[x]]

$\sin[1 - x^{0.5}]$

Domain: x >= 0
Graphed in red below

g[f[x]]

$1 - \sin[x]^{0.5}$

Domain: n <= x <= (n+1)*pi, n = 0, 2, 4, 6, . . .
Graphed in purple below

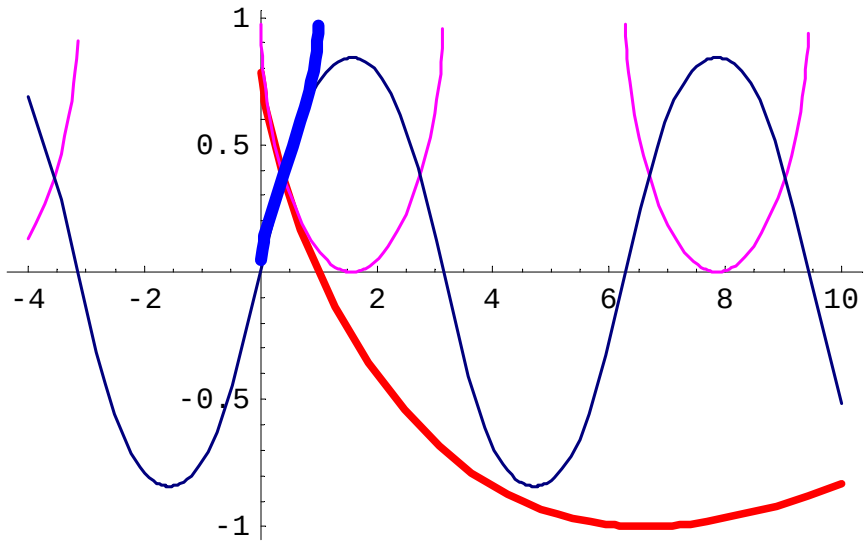
f[f[x]]

Sin[Sin[x]]
Domain: all real numbers
Graphed in dark blue below

g[g[x]]

$1 - (1 - x^{0.5})^{0.5}$

Domain: 0 <= x <= 1
Graphed in blue below



10. Given $f(x) = 1 - 3x$ and $g(x) = 5x^2 + 3x + 2$, find the functions $f \circ g$, and $g \circ f$, and state their domains.

f[g[x]]

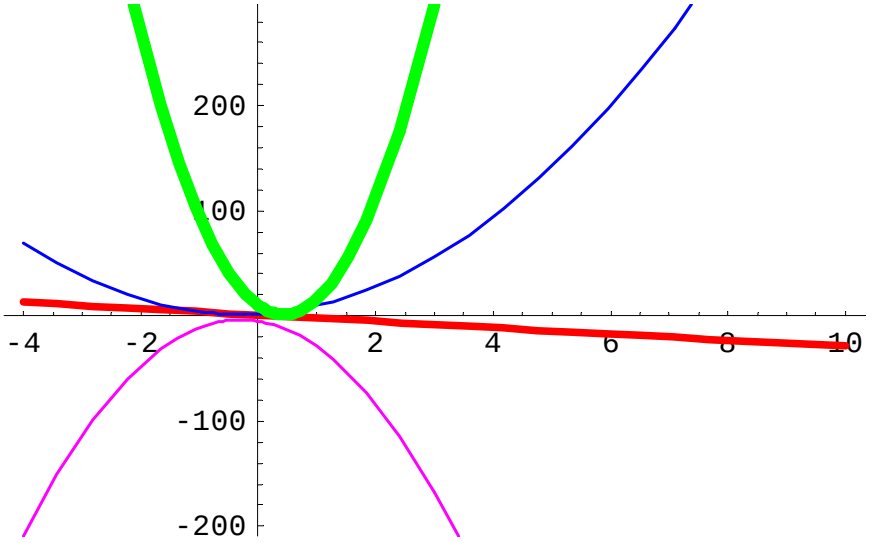
$1 - 3(2 + 3x + 5x^2)$

Domain: All real numbers
Graphed below in purple

g[f[x]]

$2 + 3\,(1 - 3\,x) + 5\,(1 - 3\,x)^2$

Domain: All real numbers
Graphed below in green



11. Given $f(x) = \sqrt{x-1}$, $g(x) = x^2 + 2$, and $h(x) = x + 3$ find the functions $f \circ g \circ h$, and state its domain.

f[g[h[x]]]

$(1 + (3 + x)^2)^{0.5}$

Domain: All real numbers
Graphed below in green