Logarithms Solution

Laws of logarithms:

1. *if* $\log_a(a^x) = y$ *then* $a^y = x, y = \frac{\ln x}{\ln a}$	2. $\log_a x = \frac{\ln x}{\ln a}$, $\log_e x = \ln x$
3. $\ln(e^x) = x, \ln e = 1$	4. *if* $\ln x = y$, *then* $e^y = x$
5. $\log_a(xy) = \log_a x + \log_a y$	6. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
7. $\log_a x^r = r \log_a x$	

1. Using the laws of logarithms, what are the following equivalent to?

a. $\log_b(x) - \log_b(y) = \log_b(\frac{x}{y})$

b. $\log_b(u) + \log_b(v) = \log_b(u * v)$

c. $\log_a y^2 + \log_a x^3 = \log_a(y^2 x^3)$

d. $\log_a y^3 + \log_a x^4 = \log_a(y^3 x^4)$

e. $\ln(e) = 1$

f. $\ln a + \frac{1}{2}\ln b = \ln(a\, b^{1/2})$

g. $2\ln 4 - \ln 2 = \ln 8$

h. $\ln x + a \ln y - b \ln z = \ln(\frac{xy^a}{z^b})$

2. Solve the following for x using known logarithm properties.

a. $1\ln x = 2$
$e^{\ln x} = e^2$
$x = e^2$

b. $\ln x = -1$
$e^{\ln x} = e^{-1}$
$x = 1/e$

c. $\ln x = 5$

$e^{\ln x} = e^5$
$x = e^5$

d. $13^x = 6$
$\ln (13^x) = \ln (6)$
$x \ln (13) = \ln (6)$
$x = (\ln 6)/(\ln 13)$

e. $7 = 2^x$
$\ln 7 = \ln 2^x$
$\ln 7 = x \ln 2$
$x = (\ln 7)/ (\ln 2)$

f. $9^x = 4$
$\ln (9^x) = \ln (4)$
$x \ln (9) = \ln (4)$
$x = (\ln 4)/(\ln 9)$

g. $e^{5-3x} = 10$
$\ln (e^{5-3x}) = \ln 10$
$5 - 3x = \ln 10$
$-3 x = \ln 10$
$x = (\ln 10)/-3$

h. $(2-\ln x)\ln x = 0$
$2 - \ln x = 0$
$2 = \ln x$
$e^2 = e^{\ln x}$
$e^2 = x$

i. $2 \ln x = 1$
$\ln x = (1/2)$
$e^{\ln x} = e^{(1/2)}$
$x = e^{(1/2)}$

j. $e^{-x} = 5$
$\ln (e^{-1x}) = \ln (5)$
$-1x = \ln (5)$
$x = -1 \ln 5$

k. $e^{2x+3} - 7 = 0$
$e^{(2x+3)} = 7$
$\ln (e^{(2x+3)}) = \ln 7$
$2x + 3 = \ln 7$
$2x = (\ln 7) - 3$
$x = (\ln 7)/2 \ - 3/2$

l. $\ln(5-2x)=-3$

$e^{\ln(5-2x)} = e^{(-3)}$

$5 - 2x = e^{(-3)}$

$-2x = e^{(-3)} + 5$

$x = (e^{(-3)})/-2 - 5/2$

m. $2^{x-5} = 3$

$\ln(2^{x-5}) = \ln 3$

$(x-5)\ln 2 = \ln 3$

$x - 5 = (\ln 3)/(\ln 2)$

$x = (\ln 3)/(\ln 2) + 5$

n. $\ln x + \ln(x-1) = 1$

$\ln(x^2 - x) = 1$

$e^{(\ln(x2-x)} = e^1$

$x^2 - x = e$

$x^2 - x - e = 0$

$x = \frac{1}{2}\left(1 + \sqrt{1+4e}\right)$

3. Find the value of each expression.

a. $\log_2 64 = 6$

b. $\log_6 \frac{1}{36} = -2$

c. $\log_8 2 = 1/3$

d. $\ln e^{\sqrt{2}} = \sqrt{2}$

e. $\log_{10} 1.25 + \log_{10} 80 = 2$

f. $\log_5 10 + \log_5 20 - 3\log_5 2 = 2$

g. $2^{(\log_2 3 + \log_2 5)} = 15$

h. $e^{3\ln 2} = 8$