

## **REQUIRED MATHEMATICAL SKILLS**

Cadets enter the United States Military Academy (USMA) with different backgrounds in mathematics. To successfully begin work in math/science/engineering (MSE) courses at USMA, cadets must arrive with knowledge of certain mathematical skills and concepts. The MSE Committee deems the items below to be fundamental concepts that all entering cadets must possess. Early in Cadet Basic Training, the Department of Mathematical Sciences administers a “Gateway” exam to assess fundamental concepts. Therefore, we encourage incoming cadets to evaluate their mathematics skills and work at remediation of any deficiencies prior to arriving at USMA.

The Department of Mathematical Sciences administers two additional “Gateway” exams during the first semester in MA103 (Mathematical Modeling and Introduction to Calculus). Self-paced texts and software files offer opportunities for self-remediation. Failure to pass at least one “Gateway” exam at mastery level ( $\geq 80\%$ ) may result in placement in MA100 (Precalculus) during the second semester instead of continuing into MA104 (Calculus and Differential Equations), or other scheduled remediation. Cadets enrolled in MA100 who do not demonstrate proficiency in these fundamental mathematics skills by the end of their first year may be separated from USMA.

Almost any high school algebra book is a suitable reference for these fundamental concepts.

**Note: All calculations should be performed without the use of technology (i.e., calculator). Some examples of skills are provided in parentheses.**

### **1. Algebra and Real Numbers.**

- a. Use symbols and operators to represent ideas and objects and the relationships existing between them.
- b. Understand the relationship between measures of the physical world. (Velocity, distance and time: On a 40-mile car trip to Middletown, NY, you drive the first twenty miles at 40 mph and the last twenty miles at 60 mph. What is your average speed during the trip?)
- c. Know and apply the following algebraic properties of the real number system: identity, associative, commutative, inverse, and distributive.
- d. Express numbers using scientific notation. (Express 0.004312 in scientific notation.)

### **2. Radicals and Exponents.**

- a. Convert between radical and rational exponent form. (Transform  $\frac{1}{\sqrt{x+2}}$  to the rational exponent form  $(x+2)^{-\frac{1}{2}}$ .)
- b. Manipulate algebraic expressions that contain integer and rational exponents. (Simplify  $4^{\frac{3}{2}} \cdot 27^{\frac{2}{3}}$ .)

### **3. Algebraic Expressions.**

- a. Add, subtract, multiply, and divide algebraic expressions. (Find the remainder when  $x^3 - 7x^2 + 9x$  is divided by  $x - 2$ .)
- b. Simplify algebraic expressions. (Expand and simplify  $(x-3)(x-2)(x-1)$ .)

### **4. Factoring / Prime Numbers**

- a. Write a number as the product of factors. (Write 42 as the product of prime factors.)
- b. Solve for the roots of a polynomial by factoring. (Find the roots of  $x^2 - 5x + 6 = 0$ .)

### **5. Linear Equations, Inequalities and Absolute Values.**

- a. Solve 2 simultaneous linear equations by graphing and by substitution. (Use a graph to estimate the point of intersection of the lines  $2x + 3y = 7$  and  $-x + y = 4$ . Verify your result using back substitution.)
- b. Solve linear equations and inequalities [graphically and algebraically]. (Solve  $5(3-x) > 2(3-2x)$  for  $x$ .)
- c. Solve linear equations and inequalities with absolute values. (Solve  $|x - 4| \geq 3$  for  $x$ .)

### **6. Polynomials and Rational Inequalities.**

- a. Solve simple polynomial inequalities. (Solve  $x^2 + 3x + 6 > x - 4$  for  $x$ .)
- b. Solve simple rational inequalities. (Solve  $\frac{x-3}{x+1} < 2$  for  $x$ .)

## 7. Straight Lines.

- Determine the equation of a line. (Find the equation of a straight line passing through the points (2, 1) and (5, 4).)
- Determine the equation of a line that is parallel or perpendicular to a given line. (Find the equation of a line parallel to the line  $2y - 3x = 7$  and passing through the point (1, 2).)

## 8. Functions.

- Identify the independent and dependent variables of a function.
- Determine the domain and range of a real valued function. (Find the domain and range of the real valued function  $g(x) = \frac{1}{x^2 - 2}$ .)
- Evaluate a function at a point. (Given  $f(x) = 3x^2 - 2x + 4$ , find  $f(2a)$ .)
- Evaluate composite functions. (Given  $h(r) = 3r^2$  and  $g(s) = 2s$ , find  $h(a+2) - g(2a)$ .)

## 9. Quadratic Equations and Systems

- Solve for real and complex roots using the quadratic formula. (Find the roots of  $3x^2 + 2x = -1$ .)
- Solve a system of quadratic equations in 2 variables by substitution. (Solve the system  $y = 3 - x^2$  and  $y = 4 + 2x^2 - 2x$ .)

## 10. Trigonometric Functions.

- Define each of the 6 trigonometric functions ( $\sin(\theta)$ ,  $\cos(\theta)$ ,  $\tan(\theta)$ ,  $\cot(\theta)$ ,  $\sec(\theta)$ ,  $\csc(\theta)$ ) in terms of the sides of a right triangle. ( $\cos(\theta) = x/r$  where  $x$  is the adjacent side and  $r$  is the hypotenuse.)
- Define each of the 6 trigonometric functions in terms of  $\sin(\theta)$  and  $\cos(\theta)$ . ( $\tan(\theta) = \frac{\sin \theta}{\cos \theta}$ .)
- Know the domain and ranges for the sine, cosine, and tangent functions.
- Convert angle measures between degrees and radians. (Write 120 degrees as a radian measure.)
- Memorize and use the 30/60/90 and 45/45/90 degree reference triangles.
- Know and apply the trigonometric identity  $\sin^2(\theta) + \cos^2(\theta) = 1$ . (Simplify the expression  $2 \cos^2(\theta) + \sin^2(\theta) - 1$ .)

## 11. Logarithmic and Exponential Functions.

- Know the relationship between logarithm and exponential functions [ $y = \log_a x$ ,  $a > 0$ ,  $a \neq 1$ , is the inverse of the function  $y = a^x$ ;  $\log_a x = y \Leftrightarrow a^y = x$ ]. (Evaluate  $\log_3 27$ .)
- Know the properties of the logarithmic and exponential functions and use them to simplify logarithmic expressions. (Express as a single logarithm:  $.5 \log_{10} x - \log_{10} y$ .)
- Solve simple logarithmic and exponential equations. (Solve the equation  $3^{x+4} = 4$  for  $x$ .)

## 12. Graphs and Graphing.

- a. Graph equations and inequalities. (Sketch a graph of the function  $f(x) = 3x^2 - 2x + 7$  for  $1 < x < 5$ .)
- b. Properly label a graph (axes, intercepts, asymptotes, and roots).
- c. Know the general characteristics and shapes of the graphs of polynomial, logarithm, exponential and trigonometric functions.
- d. Transform the graph of a known function. (From the graph of  $f(x)$ , graph  $g(x) = 2f(x) - 3$ .)

## 13. Analytic Geometry.

- a. Know and apply the distance formula between 2 points. (Find the distance between the two points A(1,2) and B(-5,-3).)
- b. Know and apply the circumference and area formulas for circles, triangles, and rectangles. (If you double the radius of a circle, what happens to its circumference?)
- c. Know and apply the surface and volume formulas for cylinders, spheres and rectangular solids.
- d. Know the relationship between similar triangles. (A rectangle with base  $x$  and height 5 is inscribed in an isosceles triangle with base 10 and height 20. Determine  $x$ .)
- e. Know and apply the Pythagorean Theorem to simple geometric problems. (Given a rectangle that is 4 ft by 7 ft determine the length of the diagonal.)