An Introduction to *Mathematica*: Calculus Operations

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Derivatives

If we want to take the derivative of an expression, we can use **D[***expression*, *variable***]**

```
In[1]:= D[x^2 - 4 x + 7, x]
```

However, if we are using a function, we can use the **f'[x]** notation.

```
In[2]:= g[x_] := x^2 - 4 x + 7
        g'[x]
```

One of the uses of calculus is to determine the extreme points of a curve. While a graph can be used to estimate the highest and lowest points, calculus can be used to determine the values to any desired degree of accuracy.

Suppose you want to determine the maximum and minimum values of the the following equation

```
In[4]:= f[x_] := x^3 - 15 x + 8
```

The first step is to graph the equation and get an estimate of the extrema. After the equation has been plotted, click on the graph and move the mouse to the maximum point and CMD click. In the lower left corner of the screen, you will see the coordinates of the point where you clicked. You should get a y value of approximately 30.

```
In[5]:= Plot[f[x], {x, -6, 6}];
```

To find the extreme points, take the deriviative and set it equal to zero. Notice the == sign inside of **Solve**.

```
In[6]:= critpoint = Solve[f'[x] == 0, x]
```

Decimal values can be obtained using the **N[]** or **//N** commands.

```
In[7]:= N[critpoint]
        critpoint // N
```

We can verify that these are solutions by substituting back into the derivative using **/.**

```
In[9]:= f'[x] /. critpoint
```

We can find the function values by substituting into the function.

```
In[10]:= f[x] /. critpoint // N
```

To check on concavity, substitute these values into the second derivative.

```
In[11]:= f''[x] /. critpoint // N
```

Since the second derivative is negative at the first critical point, it gives a maximum point. We can redraw the graph with labels at these points. Note that **DisplayFunction->Identity** hides the graph and **DisplayFunction->$DisplayFunction** shows the graph.

```
In[12]:= mycurve = Plot[f[x], {x, -5, 5}, Axes → False,
           Frame → True, DisplayFunction → Identity]; maxtext =
          Graphics[{Text["Relative Maximum at (-2.23, 30.36)", {-4.23, 40}, {-1, 0}],
            {Hue[1], PointSize[.03], Point[{-2.23, 30.4}]}
           }];
         mintext = Graphics[
            {Text["Relative Minimum \n at (2.23, -14.36)", {-1.5, -25}, {-1, 0}],
             {Hue[0.5], PointSize[.03], Point[{2.23, -14.36}]}
            }];
```

```
In[14]:= Show[mycurve, mintext, maxtext, DisplayFunction → $DisplayFunction];
```

We can add the tangent line to the function at a point, say $x = 1.5$. Because of the easier use of functions, we can write the line immediately.

```
In[15]:= xpoint = 1.5
         myline = f'[1.5] * (x - xpoint) + f[1.5]
         mylinegraph =
           Plot[myline, {x, -5, 5}, PlotStyle → Hue[0.7], DisplayFunction → Identity];
         Show[mycurve, mintext, maxtext, mylinegraph,
           DisplayFunction → $DisplayFunction];
```

Series

Infinite series expansions are found using the **Series** command. For example, the following finds the Taylor series of $\frac{1}{1+x}$ at $x = 2$.

```
In[19]:= myseries = Series[1/(1 + x), {x, 2, 5}]
```

To remove the "Big O" notation at the end, use the **Normal** command either at the beginning or end of the **Series** command.

```
In[20]:= Normal[myseries]
         myseries // Normal
```

Sum of a Series

To get the sum of a series, use the **Sum** command

In[22]:=
```
Sum[1/x, {x, 1, 100}]
```

Integration

Integration can be done in either exact or numerical form.

In[23]:=
```
Integrate[Tan[x], {x, 0, Pi / 4}]
```

In[24]:=
```
NIntegrate[Tan[x], {x, 0, Pi / 4}]
```

You Try It!

(1) Determine the maximum and minimum points of the function $y = 3x^4 + 4x^3 - 4x^2 - 8x + 2$

(2) Create a graph of the function that has the extrema labeled on the graph.

(3) Draw the tangent line to the graph at $x = 2$.

(4) Determine the value of $\sum_{x=1}^{20} \frac{1}{x^2}$

(5) Determine the values of the following integrals.

$$\int_1^5 x^3 \, dx, \qquad \int_0^1 \frac{1}{\sqrt{9 - 4x^2}} \, dx, \qquad \int_0^{1+\sqrt{w}} \sqrt{x^2 + 9} \, dx$$