An Introduction to *Mathematica*: Graphing Implicit, Parametric, and Polar Equations

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In order to use the named colors, start by loading the following package

```
In[1]:= << Graphics`Colors`
```

In order to graph implicit equations, load the package

```
In[2]:= << Graphics`ImplicitPlot`
```

Implicit Functions

An implicit function is one in which you do not solve the equation for y. The implicitplot command will allow you to graph this type of equation. Note that the default aspect ratio is automatic, i.e. circles appear as circles.

```
In[3]:= ImplicitPlot[x^2 + y^2 == 1, {x, -2, 2}, {y, -2, 2}];
```

```
In[4]:= ImplicitPlot[x^3 + x^2 * y^3 == 4, {x, -10, 10}, {y, -10, 10}];
```

The top of the graph has been chopped off, This was unnoticable in *Maple*. Increasing the number of points and changing the *y* values helps the picture only a small amount.

```
In[5]:= ImplicitPlot[x^3 + x^2 * y^3 == 4, {x, -10, 10}, {y, -10, 15}, PlotPoints → 300];
```

Parametric Equations

Parametric equations describe the horizontal and vertical positions of a point on a curve by writing both x and y in terms of a third variable t, which often represents time. This material is usually covered in Calculus II. To graph parametric equations, define the x and y values and then use the plot command with the format **ParametricPlot[{x,y},{t,*start, end*}]**. You can use any of the options that are used with the graphs of standard functions..

```
In[6]:= xfunction = Cos[t]
        yfunction = Sin[3 t]
        ParametricPlot[{xfunction, yfunction}, {t, 0, 2 Pi}];
```

We can also simply place the *x* and *y* values inside of the plotting command.

```
In[9]:= ParametricPlot[{Cos[3 t] * Cos[t], Cos[3 t] * Sin[t]},
          {t, 0, 2 Pi}, PlotStyle → {Blue, Thickness[0.02]}, Axes → False];
```

Polar Equations

A polar coordinate system plots coordinates based on the distance of a point from the origin (the radius) and the angle formed by the x -axis and the line between the origin and the point.

There are two methods of plotting in polar coordinates. the first method requires loading the package **<<Graphics`Graphics`**. You can then use the **PolarPlot** command.

```
In[10]:= << Graphics`Graphics`

In[11]:= PolarPlot[Sin[2 theta], {theta, 0, 2 Pi}];

In[12]:= PolarPlot[{Sin[2 theta], Cos[2 theta]}, {theta, 0, 2 Pi},
          PlotStyle → {{Red, Thickness[0.02], Dashing[{0.005, 0.05}]},
            {Blue, Thickness[0.005], Dashing[{0.01, 0.02}]}}]
```

The disadvantage of using the **PolarPlot** command is that, for the most part, the standard polar functions are centered around the origin. It would require some messy algebra to move the above graph so that it is centered around another point, say (2, 3).

To avoid this difficulty, we can use parametric equations. The relationship between a polar equation r and parametric equations is that $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Consider the following. Notice that the graph is now centered around (2, 3).

```
In[13]:= r1 = Cos[2 theta]
         x1 = 2 + r1 * Cos[theta]
         y1 = 3 + r1 * Sin[theta]
         ParametricPlot[{x1, y1}, {theta, 0, 2 Pi}];
```

The graph is a bit squashed but we can fix that using the option **AspectRatio->Automatic**. We can also add other curves to the drawing.

In[34]:=
```
r1 = 2 Sin[2 theta]
x1 = -2 + r1 * Cos[theta]
y1 = 3 + r1 * Sin[theta]
firstcurve = ParametricPlot[{x1, y1}, {theta, 0, 2 Pi},
   DisplayFunction → Identity, PlotStyle → {Carrot, Thickness[0.015]}];

r2 = 2 Sin[3 theta]
x2 = 2 + r2 * Cos[theta]
y2 = 3 + r2 * Sin[theta]
secondcurve = ParametricPlot[{x2, y2}, {theta, 0, 2 Pi},
   DisplayFunction → Identity, PlotStyle → {Chartreuse, Thickness[0.02]}];

r3 = 2 Sin[6 theta]
x3 = -2 + r3 * Cos[theta]
y3 = -3 + r3 * Sin[theta]
thirdcurve = ParametricPlot[{x3, y3}, {theta, 0, 2 Pi},
   DisplayFunction → Identity, PlotStyle → {Cobalt, Thickness[0.01]}];

r4 = 1.5 Sin[5 theta]
x4 = 2 + r4 * Cos[theta]
y4 = -3 + r4 * Sin[theta]
fourthcurve = ParametricPlot[{x4, y4}, {theta, 0, 2 Pi},
   DisplayFunction → Identity, PlotStyle → {DeepPink, Thickness[0.012]}];

Show[{firstcurve, secondcurve, thirdcurve, fourthcurve}, AspectRatio →
   Automatic, DisplayFunction → $DisplayFunction, Axes → True, Frame → True];
```

You Try It!

(1) Graph the set of parametric equations $x = 5\cos(t) + 14\cos(15t)$, $y = 5\sin(t) + 14\sin(15t)$.

(2) Graph the polar function $r = \theta$ where $0 \le \theta \le 6\pi$.

(3) Graph the implicit function $|x|^{2/3} + |y|^{2/3} = 1$ on the interval $-1 \le x \le 1$, $-1 \le y \le 1$. Use 1000 points and **Abs[]** for the absolute value function.

(4) Create a single graph containing a plot of an implicit, a parametric, and a polar equation.