Introduction to *Mathematica*
Systems of Equations

Bob Bradshaw, Ohlone College

Mathematica can solve systems of equations through the use of the solve command. First, enter the equations and then use the command
`Solve[{eqn1, eqn2, eqn3}, {variables}]`. be sure to use a double equal sign for the equations.

In[1]:=
```
eqn1 = 3 x + 4 y + 5 z == 2
eqn2 = -4 x + 6 y + 15 z == 22
eqn3 = 2 x + 7 y - 5 z == 2
Solve[{eqn1, eqn2, eqn3}, {x, y, z}]
```

Out[1]= $3x + 4y + 5z == 2$

Out[2]= $-4x + 6y + 15z == 22$

Out[3]= $2x + 7y - 5z == 2$

Out[4]= $\left\{\left\{x \to -\frac{200}{113}, y \to \frac{132}{113}, z \to \frac{298}{565}\right\}\right\}$

Unlike what you did in a beginning algebra course, *Mathematica* can solve systems with more equations than unknowns.

In[5]:=
```
eqn1 = 3 x + 4 y + 5 z == 2
eqn2 = -4 x + 6 y + 15 z == 22
Solve[{eqn1, eqn2}, {x, y, z}]
```

Out[5]= $3x + 4y + 5z == 2$

Out[6]= $-4x + 6y + 15z == 22$

Out[7]= $\left\{\left\{x \to \frac{15z}{17} - \frac{38}{17}, y \to \frac{37}{17} - \frac{65z}{34}\right\}\right\}$

Mathematica can solve systems with non-constant coefficients.

In[8]:=
```
eqn1 = Sin[t] x + 4 Exp[t] y + 5 z == 2
eqn2 = -4 x + Sqrt[6] y + 15 z == 22
Solve[{eqn1, eqn2}, {x, y, z}]
```

Out[8]= $4e^t y + 5z + x\sin(t) == 2$

Out[9]= $-4x + \sqrt{6}\, y + 15z == 22$

Out[10]= $\left\{\left\{x \to \frac{2\left(\sqrt{6} - 44e^t\right)}{\sqrt{6}\sin(t) + 16e^t} - \frac{5\left(\sqrt{6} - 12e^t\right)z}{\sqrt{6}\sin(t) + 16e^t}, y \to \frac{2(11\sin(t) + 4)}{\sqrt{6}\sin(t) + 16e^t} - \frac{5z(3\sin(t) + 4)}{\sqrt{6}\sin(t) + 16e^t}\right\}\right\}$

Mathematica can also solve systems with non-linear terms.

```
In[11]:= eqn1 = x^2 + y^2 == 4
         eqn2 = x + y == 1
         Solve[{eqn1, eqn2}, {x, y}]
```

Out[11]= $x^2 + y^2 == 4$

Out[12]= $x + y == 1$

Out[13]= $\left\{\left\{x \to \frac{1}{2}\left(1-\sqrt{7}\right),\, y \to \frac{1}{2}\left(1+\sqrt{7}\right)\right\},\, \left\{x \to \frac{1}{2}+\frac{\sqrt{7}}{2},\, y \to \frac{1}{2}\left(1-\sqrt{7}\right)\right\}\right\}$

For some systems, the results are too complex to read. Use **`NSolve`** may be a better choice.

```
In[14]:= eqn1 = x^2 + y^4 == 4
         eqn2 = x + y == 1
         Solve[{eqn1, eqn2}, {x, y}]
```

Out[14]= $y^4 + x^2 == 4$

Out[15]= $x + y == 1$

Out[16]=

$$\left\{\left\{x \to 1 + \frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)} - \frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163+6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163+6\sqrt{1929}} - \frac{4}{\sqrt{\frac{1}{3}\left(-2-\frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)}}},\right.\right.$$

$$\left. y \to -\frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)} + \frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163+6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163+6\sqrt{1929}} - \frac{4}{\sqrt{\frac{1}{3}\left(-2-\frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)}}}\right\},$$

$$\left\{x \to 1 + \frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)} + \frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163+6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163+6\sqrt{1929}} - \frac{4}{\sqrt{\frac{1}{3}\left(-2-\frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)}}},\right.$$

$$\left. y \to -\frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)} - \frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163+6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163+6\sqrt{1929}} - \frac{4}{\sqrt{\frac{1}{3}\left(-2-\frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163+6\sqrt{1929}}\right)}}}\right\},$$

$$\Bigg\{x \to 1 - \frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163 + 6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)} -$$

$$\frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163 + 6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163 + 6\sqrt{1929}} + \frac{4}{\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)}}},$$

$$y \to \frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163 + 6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)} +$$

$$\frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163 + 6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163 + 6\sqrt{1929}} + \frac{4}{\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)}}}\Bigg\},$$

$$\Bigg\{x \to 1 - \frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163 + 6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)} +$$

$$\frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163 + 6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163 + 6\sqrt{1929}} + \frac{4}{\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)}}},$$

$$y \to \frac{1}{2}\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163 + 6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)} -$$

$$\frac{1}{2}\sqrt{-\frac{4}{3} + \frac{35}{3\sqrt[3]{163 + 6\sqrt{1929}}} - \frac{1}{3}\sqrt[3]{163 + 6\sqrt{1929}} + \frac{4}{\sqrt{\frac{1}{3}\left(-2 - \frac{35}{\sqrt[3]{163+6\sqrt{1929}}} + \sqrt[3]{163 + 6\sqrt{1929}}\right)}}}\Bigg\}\Bigg\}$$

```
In[17]:= NSolve[{eqn1, eqn2}, {x, y}] // ColumnForm
```

Out[17]= $\{x \to 1.27046 - 1.55623\, i,\ y \to -0.27046 + 1.55623\, i\}$
$\{x \to 1.27046 + 1.55623\, i,\ y \to -0.27046 - 1.55623\, i\}$
$\{x \to 1.85894,\ y \to -0.858944\}$
$\{x \to -0.399864,\ y \to 1.39986\}$

Linear Algebra

Mathematica also has a full set of linear algebra commands. To create a matrix, you can use either list notation or matrix notation. List notation is like the following.

```
In[18]:= bb1 = {{2, 3, 4}, {2, 5, 6}, {1, 0, 9}}
```

Out[18]= $\begin{pmatrix} 2 & 3 & 4 \\ 2 & 5 & 6 \\ 1 & 0 & 9 \end{pmatrix}$

To use the matrix notation, choose **`Input->Create Table/Matrix/Palette`** from the menus and use the mattrix option. Alternatively, you can use the keyboard shortcut CTRL SHIFT C.

In[19]:= **bb2 =** $\begin{pmatrix} 2 & 5 & 6 \\ 1 & 4 & 8 \\ 9 & 2 & 0 \end{pmatrix}$

Out[19]= $\begin{pmatrix} 2 & 5 & 6 \\ 1 & 4 & 8 \\ 9 & 2 & 0 \end{pmatrix}$

You can find the determinant of a matrix using the determinant command **`Det`**.

In[24]:= **`Det[bb1]`**

Out[24]= 34

In[25]:= **bb3 =** $\begin{pmatrix} 2 & 3 & 1 & 4 \\ 5 & 3 & 1 & 7 \\ 8 & 6 & 4 & 8 \end{pmatrix}$

Out[25]= $\begin{pmatrix} 2 & 3 & 1 & 4 \\ 5 & 3 & 1 & 7 \\ 8 & 6 & 4 & 8 \end{pmatrix}$

In linear algebra and finite math, systems of linear equations are solved using a technique called Gauss-Jordan elimination. The *Mathematica* command for this is **`RowReduce`**.

In[26]:= **`RowReduce[bb3]`**

Out[26]= $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & -2 \end{pmatrix}$

You Try It!

1. Find the solution to the set of equations
3x + 5y + 2z – 3w = 9
4x + 7y + 5z – w = 19
3x + 6y + 2z – 2w = 3
x + 9y – 8z – 13w = 29
2. Sketch the graph of the two equations x^2+y^2 = 9; and x^2/16+y^2 = 1;. Use *Mathematica* to find the intersection points.
3. Use the method of **`RowReduce`** to solve problem (1).