UNITED STATES MILITARY ACADEMY

DREDGING THE HUDSON RIVER

MA103: DISCRETE DYNAMICAL SYSTEMS

SECTION 8B

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Comment [C1]: This example write-up is a compilation of cadet projects. I would like to thank Cadets Castle, Chonowski, Knepp, Smith, Riddle, and Nash for their excellent work. Much of the write-up is verbatim from their projects. I chose not to quote their actual work since I am not trying to pass this off as my own work, rather I am using this as a guide (not the guide) to help my students excel. Sometimes you will see areas noted in red – these are corrections to the cadets’ paper.

Comment [C2]: The format used in this project is particular to this project scenario. Use this as an example not a template for success.

Comment [C3]: Format for Title page is found in the DWW.

____ MY DOCUMENTATION IDENTIFIES ALL SOURCES USED AND ASSISTANCE RECEIVED IN COMPLETING THIS ASSIGNMENT.

Comment [C4]: Acknowledgement statement added as of 13 DEC 06.

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A – PCB levels for an initial amount of 350,000 pounds and a dredging effect of 5% each year.

Documentation

Dredging the Hudson River

(Abstract)

Over a course of thirty years, and ending in 1977, 1.1 million pounds of PCBs (polychlorinated biphenyls) were dumped in the Hudson River by manufacturing companies. These contaminants are detrimental to human health if consumed in large quantities. It is now estimated that of the initial 1.1 million pounds only 300,000 remain today. Nature removes pollutants from the river at a rate of 7.5% annually, yet 6,200 lbs. are also added to the river each year. The EPA and DEC are both calling for the dredging of a 40-mile stretch of the upper Hudson in an attempt to reduce the amount added by 5.5% annually. This subject has become a point of controversy in an upcoming senate election in which Candidate "Clean" is supportive and Candidate "Spotless" is very much against the dredging process.

While the dredging plan would result in faster removal of pollutants, the difference in speed is not significant. Only a few hundred pounds of pollutants separate the dredging and natural plans during the first couple of years. After ten years, a difference of ten thousand pounds separates the two plans. Still, this amount is not significant enough to warrant the high price of dredging. Over a long enough period of time only the dredging plan removes the pollution completely, while the natural plan stops at 82,666.7 lbs (Figure 2). However, dredging for such a long period would not be economically sound. The natural plan is the better of the two.

Comment [C5]: As you progress through the curriculum, you may be asked to include an abstract. An abstract is a one-page document providing information on what the project concerns, the bottom line results and conclusions. Think of this as the Superintendent's version of your work. The Superintendent is a very busy person and does not have time to read and digest every project report that he may be responsible for. It is your job to distil your analysis into a concise document where he can get the idea of what you found. The most important component of this document is your findings. This page is not numbered and does not count to any "page budget."

Background

Comment [C6]: The background and facts (in the next section) should be summarized in your own words. They should not simply be "cut and pasted" from the main document.

The Hudson River is an essential element to the beauty surrounding the United States Military Academy and all cities that rest on its graceful valleys. But it is what lies beneath, however, that is causing a problem for those closest to and within it. The river is highly adulterated with polychlorinated biphenyls (PCBs) as a result of a thirty-year period ending in 1977 that left 1.1 million pounds of pollutants in the water and mud. 90% of these PCBs came from a forty-mile length of the northern part of the river between Fort Edward and Troy. There is an estimated 300, 000 pounds of PCBs in the river this year, and each year it is estimated that 6200 pounds are being added to the river. The consumption of fish contaminated with PCBs have the potential to cause cancer in humans in addition to liver dysfunction, digestive disorders, headaches, nausea, and fatigue.

Comment [C7]: The background is one of the most important parts of the project. It is important to provide a thorough recap of the problem. Assume that the reader was not given the information that you were given. Discuss the significance of the problem and why it is important to solve. A papers go beyond what is given and incorporate a little research.

There have been many proposed plans to relieve the river, its inhabitants, and its neighbors of this unhealthy burden. The most recent plan calls for the dredging of 2.65 million cubic yards of sediment from a forty mile stretch in the upper portion of the Hudson. Senator Candidate "Clean" claims that "dredging" the river, the process of scraping a river bottom to remove pollutants (see Figure 1), will reduce the amount of PCBs *added* each year by 5.5%.

Comment [C8]: Notice the figure is centered and labeled IAW the Little Brown Handbook. The documentation portion should not be in the title of the figure but instead should be a footnote or an endnote.

Figure 1. Photograph of environmental dredging courtesy of Keene Engineering. Retrieved September 22, 2002 from the World Wide Web: http://clearwater.org/news/dredging.html.

The natural process of self-cleansing will remove 7.5% of the original 300,00 pounds each year.

This problem has become a major problem in an upcoming senate election. The two candidates, candidate "Spotless" and candidate "Clean", are at odds on the subject. Candidate "Clean" claims that his dredging plan would reduce the amount of pollutants to 150,00 lbs in ten years. However, Candidate "Spotless" believes that Candidate "Clean's" plan is inaccurate due to its dependency on the initial

estimate of the pollution levels, and the reduction of added pollution by 5.5% annually. There is also concern that the dredging will only stir up the settled pollutants. The purpose of this report is to determine the effectiveness of the dredging process and to determine whether or not it should be used.

Facts

The facts surrounding this are centered on the PCBs and how much *has* entered, how much *will* enter, and how much will *leave* the Hudson River, and how *long* the process will take. Currently, there are 300,000 pounds of polychlorinated biphenyls in the river. Each year 6200 pounds are added despite regulatory efforts. Natural processes reduce the 300,000 pounds of PCB's *already* in the river by 7.5% each year. Senator Candidate "Clean's" plan will reduce the 6200 pounds *added* each year by 5.5%. According to Senator "Clean," this process will reduce the rivers contaminates to only 150,00 in 10 years.

Assumptions

Comment [C9]: Assumptions must be both valid (reasonable) and necessary for your mathematical model. In addition to listing assumptions, one must explain why they are both valid and necessary. For a real-world problem, problem solvers must seek to confirm or deny each assumption.

For example, the EPA might take quarterly samples of the pollution level to verify that it continues to decline at the same rate. Also, researchers will continue to monitor for new technology to improve the effectiveness of dredging.

If any assumption turns out to be untrue, it's time to adjust the model and conduct more analysis.

Assumptions are simplifications of the real world that allow the problem to be more easily solved. In order to determine the effects of the dredging operation it is thus necessary to make the following simplifying assumptions to make the model mathematically tractable.

1. The initial level of pollutants – When calculating the concentration of the 2.65 million cubic yards of sediment in the 40 mile stretch of the Hudson River Valley, certain experimental errors will occur. When taking a sample of the water to determine the PCB concentration it is assumed that the pollutants are evenly distributed over the Hudson River. In real life this assumption is extremely unlikely. Different locations in the river will likely contain varied concentrations of contaminants due to differences in water flow patterns.
2. Natural processes will reduce the pollution at a constant rate of 7.5% a year – On average, it is assumed that the natural processes of evaporation will reduce the pollution at a rate of 7.5% each year. Nature is anything but predictable. Unusual high or low water levels as well as lengthy changes in temperature may impact the natural process of pollution reduction.

3. There is a constant amount of pollution being deposited into the river each year – To simplify the problem, it is assumed that 6200 pounds of contaminant are added to the river each year. It is likely that peaks in production spurred by the economy would affect the amount of pollutant added to the river each year.
4. Funding will be available to support dredging – In the world of politics it is not uncommon to question policy and seek its change. Dredging is an expensive and labor intensive process. It is assumed that there will be adequate funding and public support to allow the operation to succeed.
5. Technology is constantly improving our lives. It is assumed that no innovations will alter the ability to decrease pollutions being added to the river.

Analysis

Comment [C10]: Depending on your instructor, this section may include the step-by-step development of your analytical solution(s). Some instructors prefer the mathematical development to be presented in an appendix.
The most important element is that the work is presented in a clear, concise, and organized manner.
Either way, it is important to present your underlying models and the major results: the analytic, graphic, and at least selected values from your numeric solution(s).

In order to accurately estimate the long term effects of the dredging process, it was necessary to develop a mathematical model of the real world problem. Since the PCB concentration in the Hudson River was changing each year, we chose to model the discrete change using a recursive function, whereby the new level of contaminant was proportional to the amount of contaminant in the river during the previous year. In developing our model the following variables are defined:

p(n) – the amount of PCB contaminant in the 40 mile stretch of the Hudson River after n years.

Comment [C11]: You should italicize all variables.

n = 0, 1, 2, … years

p(0) = 300,000 pounds

<u>Model 1</u> - In the first model we will suggest that the new level of contaminant p(n+1) is equal to the previous years (p(n)) amount minus any contaminant reduced by natural processes plus the amount assumed to be added each year.

Future = Present – Change

$p(n+1) = p(n) - .075p(n) + 6200$, thus
$p(n+1) = 0.925p(n) + 6200$

Comment [C12]: All equations should be centered and developed in Equation Editor or MathType.

Model 2 - The second model adds to the previous model an effect caused by the dredging operation. It is suggested that the dredging will cause the 6200 pounds of contaminant added each year to be reduced by 5.5% each year. Thus, initially 6200 pounds would be added and after 1 year the amount added would be (1-.055) 6200 or 0.945(6200). The second year would further reduce this amount by 5.5% causing 0.945(0.945(6200)) or $(0.945)^2(6200)$, thus an exponential term is added to the model. The variable for the second model is changed to q(n) to differentiate between the two models. q(n) still represents the amount of PCB contaminant in the 40 mile stretch of the Hudson River after n years

$$q(n+1) = 0.925q(n) + 6200(0.945)^n$$

An analytical solution is a function that results in an identity when substituted in the dynamical system. We sometimes call these types of solutions explicit because it is not necessary to know the previous years amount of pollutant to determine the amount for the new year.

Analytic solution for Model 1:

We conjecture that the solution to model 1 will take the form

$$p(k) = c(r)^k + d$$

where r represents the rate of change for the natural process of change in pollution. Thus,

$$p(k) = c(0.925)^k + d$$

To determine the constant d, we substitute the arbitrary constant d into the recursive model everywhere there exists a sequence term p(0), p(1), p(2), …, p(n).

$$p(n+1) = 0.925p(n) + 6200$$
$$d = 0.925d + 6200$$

solving for *d*

$$d = 0.925d + 6200$$
$$0.0725d = 6200$$
$$d = 82{,}666.67$$

therefore,

$$p(k) = c(0.925)^k + 82{,}666.67$$

Using the initial amount of pollutant p(0) = 300,000 it is possible to solve for the constant c in the previous formula.

$$p(0) = c(0.925)^0 + 82{,}666.67 = 300{,}000$$
$$c + 82{,}666.67 = 300{,}000$$
$$c = 217{,}333.33$$

therefore,

$$p(k) = 217{,}333.33(0.925)^k + 82{,}666.67$$

<u>Analytic solution for Model 2:</u>

We conjecture that the solution to model 2 will take the form

$$q(k) = c(r)^k + d(s)^k$$

where r represents the rate of change for the natural process of change in pollution and s represents the rate of change for the dredging process. Thus,

$$q(k) = c(0.925)^k + d(0.945)^k$$

To determine the constant d, we rewrite the conjecture $d(0.945)^k$ in terms of n and n+1 and substitute them into the original recursive model.

$$q(n+1) = 0.925q(n) + 6200(0.945)^n$$
$$d(0.945)^{n+1} = 0.925[d(0.945)^n] + 6200(0.945)^n$$

$$q(k) = d(0.945)^k$$
$$q(n) = (0.945)^n$$
$$q(n+1) = (0.945)^{n+1}$$

Since all terms have $(0.945)^n$ we separate $d(0.945)^{n+1}$ into $0.945d(0.945)^n$

$$0.945d(0.945)^n = 0.925d(0.945)^n + 6200(0.945)^n$$

dividing through $(0.945)^n$

$$0.945d = 0.925d + 6200$$

solving for *d*

$$0.945d = 0.925d + 6200$$
$$0.02d = 6200$$
$$d = 310{,}000$$

therefore,

$$q(k) = c(0.925)^k + 310{,}000(0.945)^k$$

Using the initial amount of pollutant $q(0)$ = 300,000 it is possible to solve for the constant c in the previous formula.

$$q(0) = c(0.925)^0 + 310{,}000(0.945)^0 = 300{,}000$$
$$c + 310{,}000 = 300{,}000$$
$$c = -10{,}000$$

therefore,

$$q(k) = -10{,}000(0.925)^k + 310{,}000(0.945)^k$$

Results

Comment [C13]: Here is where you answer the questions that were posed to you.

The results of the two different models were very much alike despite their advertised difference.

Table 1 shows the pollution levels over a sixteen year period:

Table 1

Pollutant levels of model 1 (no dredging) and model 2 (dredging)

Comment [C14]: Table reference and title should be centered and on the same line. Common convention is to put all table and figure references below that table and figure.

Comment [C15]: Center tables and figures (graphs) in the body of your report and label them in accordance with the Little Brown Handbook.
This allows you to refer to them in the body of your report without ordinal directions to the reader (e.g. "…in the table below…").

Years	No Dredging	Dredging
0	300000.00	300000.00
1	283700.00	283359.00
2	268622.50	267643.83
3	254675.81	252802.78
4	241775.13	238787.03
5	229841.99	225550.52
6	218803.84	213049.75
7	208593.55	201243.70
8	199149.04	190093.60
9	190412.86	179562.88
10	182331.90	169617.02
11	174857.00	160223.42
12	167942.73	151351.32
13	161547.02	142971.67
14	155631.00	135057.06
15	150158.67	127581.58
16	145096.77	120520.81

The advertised exponential decrease in pollution does not exist. After 1 year there is only a difference of a few hundred pounds of pollution. At the ten year mark the difference grows to about twenty thousand, which in comparison to the overall problem is a very insignificant amount. The mathematical model predicts that the amount of PCB contaminant will drop below 150,000 pounds between the 12th and 13th year if dredging occurs and between the 15th and 16th years if dredging is not used.

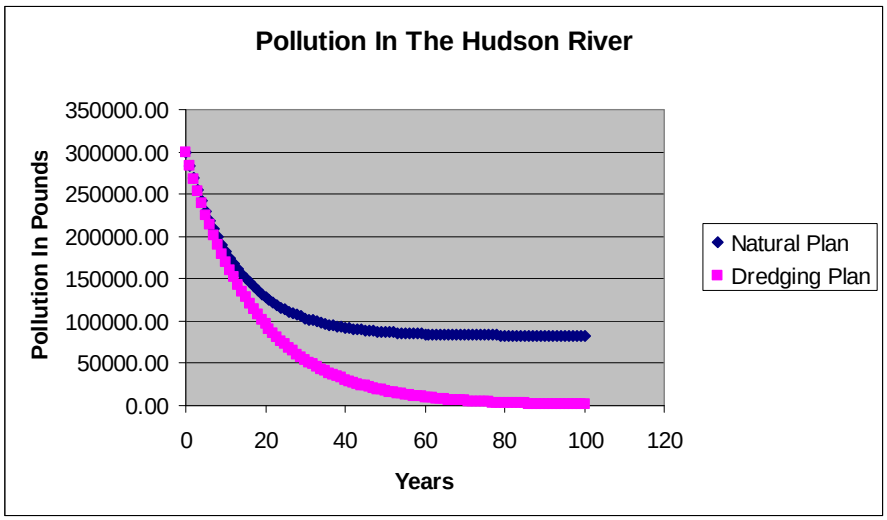

Figure 2. Graphical representation of the pounds of pollutants in the Hudson using the Natural and Dredging plans.

In the long term, PCB levels predictions for model 1, the plan reliant only on natural processes, will slowly reduce and approach a level of 82,6666 pounds. At this stage the amount of PCB added to the river each year is equivalent to the amount that natural processes are able to remove. Mathematically, this value is called the equilibrium value. PCB predictions for model 2, which includes dredging, slowly approach a level of no contaminant; however it takes slightly over 100 years to reduce to less than 1000

pounds of contaminant. It is unrealistic to believe that predictions made that far out still adhere to the model's assumptions.

The 21st Century is a period where businesses can no longer afford to pollute the environment. The state's environmental regulatory agencies now monitor how corporations and civilians treat the environment. These agencies handle clean-up projects as well as penalize those who break the law. In order to prevent further pollution that will hamper clean-up efforts the state should enforce the laws which it has set forth to protect nature. This strategy is more than feasible since it is known that the river has been cleansing itself at a natural rate since 1977, the same time when various state offices became watchdogs to protect the environment. The local population should be more willing to aid in effort to prevent further pollution of the Hudson River.

Discussion

Through the data collected, Senator "Clean's" plan does reduce the amount of pollution in the Hudson River. Senator "Clean's" plan calls for an expensive dredging process which decreases the amount of pollution exponentially. The data shows that Senator "Clean's" plan of reducing the amount of pollution in the river to 150,000 pounds is feasible in just over 10 years. The plan also decreases the amount of pollution added which speeds up the process over the natural "let nature take its course." Yet, Senator "Spotless" does bring up many points which support his theory that Senator "Clean's" plan is highly speculative. It is based on the assumptions that the initial amount of PCB contaminant condition of 300,000 pounds of pollutants is exact and that the dredging process is accurate. The plan does not take into account water evaporation, heavy rains causing an increase rise in water levels or that the amount of pollution added each year is exactly 6200 pounds. The plan also does not work exactly as Senator "Clean" proposes within his given timeframe. The pollution level does not drop below 150,000 pounds until during the 12th year of the Senators plan rather than the 10th as he proposed. According to Senator "Clean's" plan the pollution level in the river using his dredging process will reach an equilibrium state. This means the river will never truly be devoid of all pollutants under the Senators plan. If the initial

conditions were different or another dredging factor were used the Senators plan would take a lot longer than proposed. For example, if the initial amount of pollutant was determined to be 350,000 pounds of pollutants and the dredging factor was just 5 percent per year, the Senator would not meet his conditions until more than 20 years rather than 10 (See Appendix A). The plan that Senator "Clean" proposes is very dependent on his initial conditions and we have proven that if his conditions were changed slightly then his 10 year plan would be completely ineffective.

Comment [C16]: "A" papers go beyond what is asked to determine what would happen if the assumption were not correct.

Conclusions and Recommendations:

Our recommendation for the project is to inform the Senator to modify his plan in order to make it feasible for cleaning the river to the levels needed to meet his criteria. The Senator would have to adjust his plan and account for the fact that he might not have the exact initial conditions he proposed. Even with those initial conditions that he himself proposed his plan takes longer than 10 years to meet the criteria of 150,000 pounds of pollution left in the river. The Senator would first have to ensure that the level of pollution in the river would be exact and that the dredging process works as effectively as designed. The Senator would then have to find another method of cleaning the river which would keep the amount of pollution at a desired level. The level of pollution, using the current plan, would have an equilibrium at 82,666 pounds of pollution. If the Senator were to invest in a more cost effective plan he could possibly lower that equilibrium or even completely clean the river if no new pollutants were introduced. We also ask the Senator to investigate every contingency that would prevent his plan from working. Contingencies range from rising water levels to due rain fall, lowered water levels due to evaporation and so forth.

Appendix A PCB levels for an initial amount of 350,000 pounds and a dredging effect of 5% each year.

n	a(n)
0	350,000
1	336,750
2	323,929
3	311,528
4	299,538
5	287,950
6	276,756
7	265,945
8	255,508
9	245,435
10	235,718
11	226,346
12	217,310
13	208,600
14	200,207
15	192,122
16	184,335
17	176,837
18	169,620
19	162,674
20	155,991
21	149,562
22	143,380
23	137,435
24	131,720
25	126,228

Comment [C17]: If you are using endnotes, a listing of them would be presented here, followed by your Notes and Works Cited Page. If you are using footnotes, then just the Notes and Works Cited Page would go here.