Block 1 Objectives

Block I Objectives

1. Develop a mathematical model given information on how something changes.
   (a) Articulate the assumptions required for mathematical modeling.
   (b) Understand when to make an appropriate assumption.
2. Understand and use $\sum$ and Δ Notation.
3. Estimate the "area" in a general region numerically. (Accumulation Process)
   (a) Employ techniques using rectangles and trapezoids to estimate "area".
   (b) Be able to determine an upper and lower bound for an approximation of a value in a region when appropriate.
   (c) Understand the concept of "area" in a region as the limit of the sums of areas of rectangles.
   (d) Improve estimations by increasing the number of subintervals.
   (e) Understand that the "area under a curve" can take on many physical meanings. (distance, population, money gained, money lost, etc.)
4. Understand the definition of a definite and indefinite integral.
   (a) Find the antiderivatives for polynomials, exponential, rational and trigonometric functions (with and without technology).
   (b) Evaluate (with and without technology) the following definite or indefinite integrals (implies substitution):

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \qquad \int \frac{1}{u} du = \ln(|u|) + C \qquad \int e^u du = e^u + C$$

$$\int \sin(u) du = -\cos(u) + C \qquad \int \cos(u) du = \sin(u) + C$$

   (c) Understand how the accumulation process and the definite integral are related.
   (d) Understand the properties of the integral.
   (e) Given initial conditions and a function for acceleration, find velocity and position functions.
   (f) Employ the integration technique of substitution.
5. Understand and apply the Fundamental Theorem of Calculus
   (a) Understand the significance of both parts of the Fundamental Theorem of Calculus.
   (b) Understand and be able to use the Net Change Theorem as it applies to area, concentration, mass, population, cost, and distance traveled.

6. Apply integration in solving problems.
   (a) Become familiar with the concept of work in terms of a force applied over a distance.
   (b) Be able to solve lifting and slice problems.
   (c) Be able to determine total population based on rates of growth.
   (d) Be able to determine total profit(loss) given an initial startup cost and a rate of change in revenue(cost).
   (e) Be able to determine the total displacement and total distance based on velocity.
   (f) Be able to determine probabilities of certain events based on a probability density function.

   (a) Find the position vector function $\vec{r}(t)$ when the acceleration vector function $\vec{a}(t)$ and initial conditions $\vec{v}(0)$, $\vec{r}(0)$ are known.
7. Understand what a parametric equation is.
   (a) Find parametric equations for linear, parabolic, or elliptical curves.
   (b) Understand what a space curve is and how it is related to parametric equations.
8. Solve problems involving vector valued functions.
   (a) Determine the domain of a vector valued function.
   (b) Understand the domain elements are real numbers but the range elements are vectors i.e. $\mathbf{R}^1 \to \mathbf{R}^n$.
   (c) Determine the magnitude of a vector valued function.
   (d) Calculate the indefinite integral of a vector valued function.
   (e) Calculate the definite integral of a vector valued function.
9. Model and analyze aspects of projectile motion.
   (a) Derive the equations of projectile motion.
   (b) Determine maximum range.
   (c) Determine angles of elevation.
   (d) Determine muzzle velocity.
   (e) Speed at impact.
   (f) Ability to hit desired target.
10. Understand and calculate arc lengths.
11. Understand the relationship between acceleration, velocity, speed, and position as functions of time.

Block I Goal Problems

By the end of this block of instruction, you should be very comfortable analyzing problems similar to these:

1. When we estimate distances from velocity data it is sometimes necessary to use times $t_0, t_1, t_2, t_3, \ldots$ that are not equally spaced. We can still estimate distances using the time periods $\Delta t_i = t_i - t_{i-1}$. For example, on May 7, 1992, the space shuttle Endeavor was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use the data to estimate the height above Earth's surface of the space shuttle Endeavor, 62 seconds after liftoff.

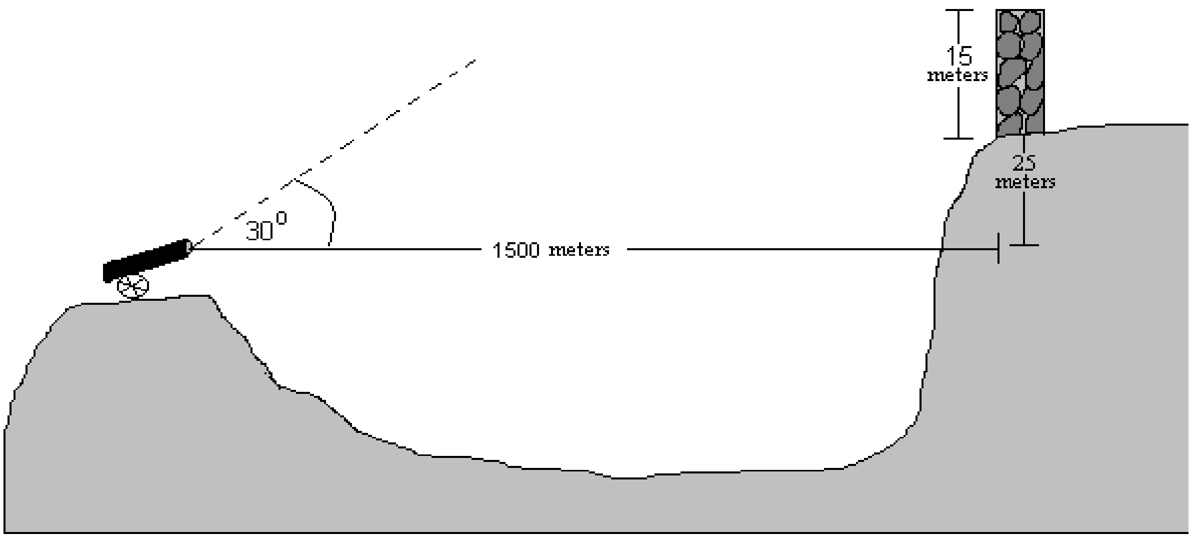
Event	Time(s)	Velocity (ft/s)
Launch	0	0
Begin Roll Maneuver	10	185
End roll Maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

2. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side? (Use the fact that water weighs $62.5 \frac{lb}{ft^3}$)

3. The marginal cost function $C'(x)$ is defined to be the rate of change of the production cost function. If the marginal cost of manufacturing x units of a product is $C'(x) = 0.0006x^2 - 1.5x + 8$ (measured in dollars per unit) and the fixed start-up cost is $C(0) = \$1,500,000$, find the cost of producing the first 2000 units and then determine the particular cost function that describes the cost of producing any amount of items, $C(x)$, for this company, based on its start up cost.

4. A hot, wet summer is causing a mosquito population explosion in a lake resort area. The number of mosquitos is increasing at an estimated rate of $2200 + 10e^{0.8t}$ per week (where t is measured in weeks). By how much does the mosquito population increase between the fifth and ninth weeks of summer? Determine a function that models the total mosquito population at any week. How many mosquitoes will there be at the end of the summer?

5. A 19th century artillery battery has established its firing position on some high ground 1500m away from an enemy position which is located on a hill top 25m higher in elevation than the battery's location. Additionally, a 15m high wall protects the enemy unit. For the following problems assume a constant acceleration due to gravity of $9.81 \frac{m}{sec^2}$.



(a) Determine a vector function in terms of the initial velocity v_o which represents the location of a cannon ball shot from the battery location at time t. Be sure to clearly identify the origin of the coordinate system you are using.

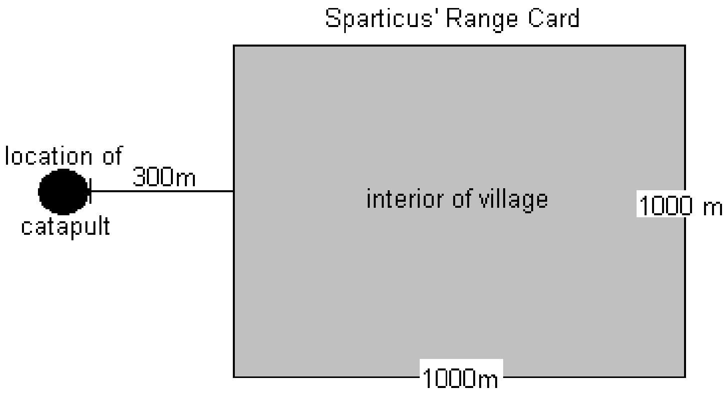
(b) Assuming the guns of the battery have a muzzle speed of $135\,\frac{m}{sec}$ and the guns are prepared to fire at an initial angle of $30°$, use your vector function to determine whether or not the cannon balls will clear the wall.

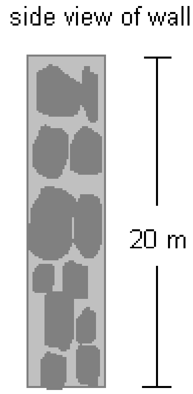
(c) Again assume that the guns of the battery have a muzzle speed of $135\,\frac{m}{sec}$ and the guns are prepared to fire at an initial angle of $35°$. Assuming the cannon balls clear the wall, what will the speed of the cannon balls be the instant they impact the ground?

(d) What is the maximum range of the cannon? (given the same muzzle speed as in part c)

(e) If the projectile does impact at the cannon's maximum range with a parabolic trajectory, what is the total distance the cannon ball traveled through the air?

6. The year is 27 AD, you and your buddy, Sparticus, are placing a village, in Gaul, under siege. A stone wall that is $20m$ high surrounds the village. You can get no closer than $300m$ to the wall. Sparticus notices that all of the buildings in the village are made of wood and thinks it would be a good idea to catapult heated rocks over the wall into the village to start it burning and end the siege. You have a catapult that generates an initial velocity of $150\frac{m}{s}$. Sparticus wants you to tell the men at what angle to set the catapult so that the rocks land in the village. Note that the village is perfectly square with each of the four walls being $1000m$ long. Find the range of angles that you can give to the men to insure success. Concern yourself only with minimum and maximum ranges. Assume you are centered on one wall and your rock will travel in a direction perpendicular to the wall.(See Figure Below)





7. A tank moves from grid location WL43542335 to grid location WL53621724 over flat terrain. All distances are measured in meters. The average speed of the tank is $5\frac{km}{hr}$.

   (a) Graph the path of the tank. Indicate direction of motion.

   (b) Develop a set of parametric equations that describes the tank's movement.

   (c) What are the coordinates of the tank 2 minutes after it starts moving?

   (d) If the tank's horizontal (east/west) location is 5000, what is it's vertical location?

   (e) Assuming the tank continues to move in the same direction and at the same speed, what will the coordinates of the tank be 1 hour later? (assume the location is on the same map sheet)