MA205 - Integral Calculus
Lesson 17: Parametric Equations

Mechanics Based Problems

1. Sketch the following curve by hand and then use the parametric plot command in *Mathematica*. Indicate with an arrow the direction in which the curve is traced as t increases.

   (a) $x = 1 + \sqrt{t}\ ,\ y = t^2 - 4t,\ \ 0 \le t \le 5$

   (b) $x = 5\sin t,\ y = t^2,\ \ -\pi \le t \le \pi$

2. Plot the following parametric equations and indicate with an arrow the direction in which the curve is traced as the parameter increases. Determine a Cartesian equation that describes the same curve.

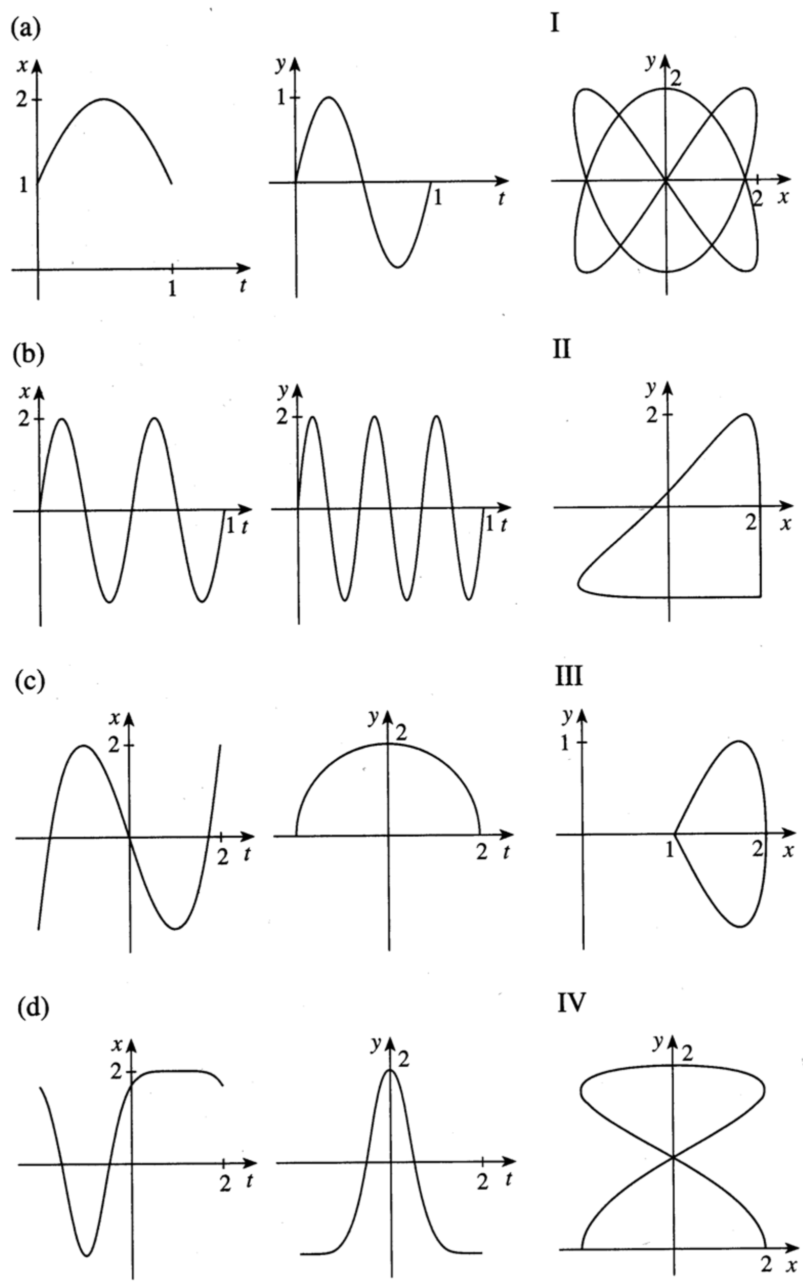
   (a) $x = \sin\theta,\ y = \cos\theta,\quad 0 \le \theta \le \pi$

   (b) $x = 4\cos\theta,\ y = 5\sin\theta,\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

   (c) $x = 1 + 2s,\ y = 1 - 3s,\quad -5 \le s \le 5$

   (d) $x = 1 + 3r,\ y = 3 + 2r,\quad -2 \le r \le 3$

3. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)-(d) with the parametric curves labeled I-IV. Give reasons for your choices.



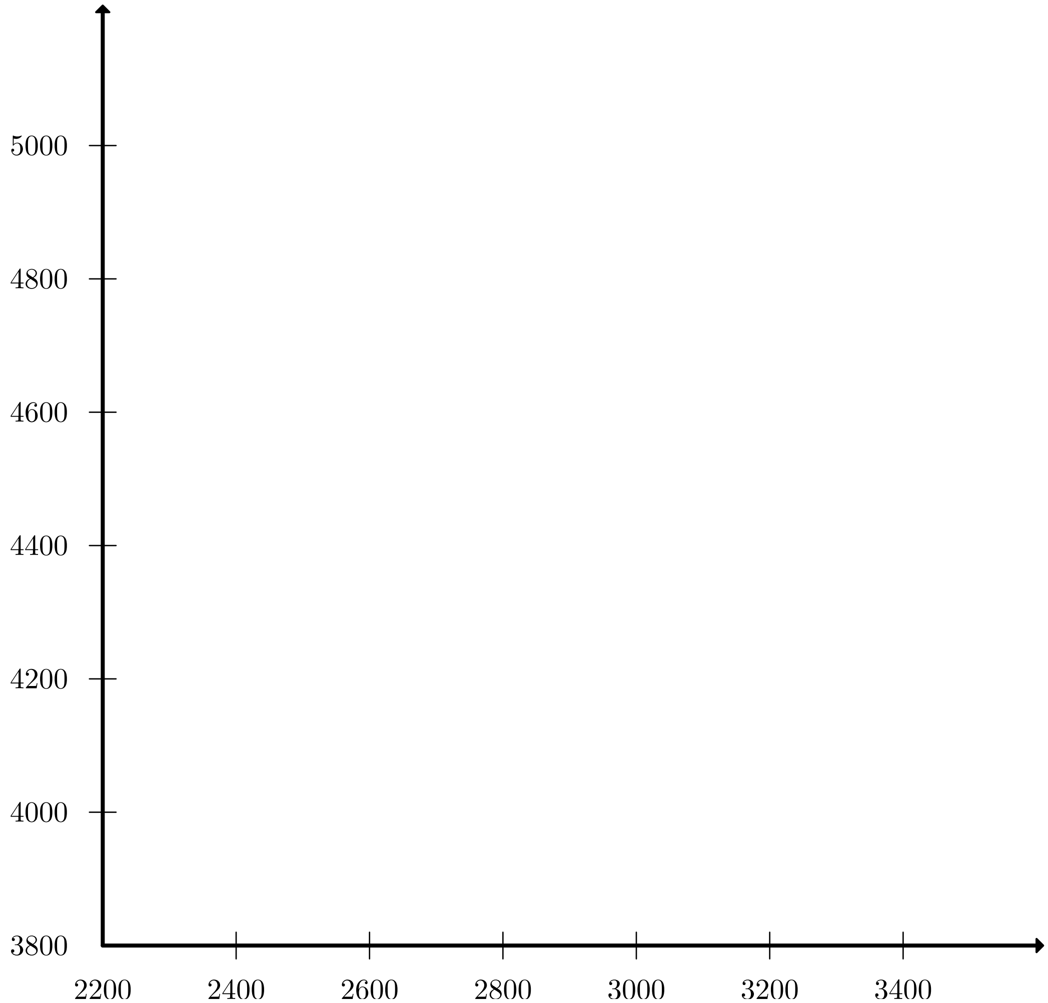
Problem Solving Problems

1. Find parametric equations for the path of s particle that moves along the circle $x^2 + y^2 = 4$ in the manner described:

   (a) Once around clockwise, starting at $(2, 0)$.

   (b) Twice around counterclockwise, starting at $(0, 2)$.

2. A tank moves from grid location WL 2230 4450 to grid location WL 3300 3950. All distances are measured in meters. The average speed of the tank is 8 km per hour. (a) On the coordinate axes provided below, graph the path of the tank. Indicate the direction of travel.



(b) How long does it take the tank to get to the final grid location? State the time in minutes.

Next you will develop Parametric Equations that model the movement, over time, of the tank. The tank moves South-East from a start location to a final location. In essence, the tank's movement has a "vertical" (North-South) component and a "horizontal" (East-West) component. A set of parametric equations will describe the different components of the total movement, one equation for each component of movement.

HORIZONTAL MOVEMENT: Let $x(t)$ represent the horizontal location of the tank at some time t, measured in minutes.

(i) What is the horizontal distance (in meters) traveled by the tank?

(ii) What is the average speed (in meters per minute) of the tank in the horizontal direction? Recall that average speed is just total displacement over total time. We denote this value as v_x or velocity in the x direction.

(iii) If the tank starts at the horizontal coordinate of 2230 (this is x_o), and travels at the speed found above, for the total time found in question b, what should be his final horizontal coordinate?

(iv) A general expression for the horizontal movement of the tank can be written as: $x(t) = x_0 + (v_x)t$. Check the units of measure involved in this equation. Does it make sense? Create a specific expression by substituting in the known value for the initial horizontal location and the horizontal velocity.

VERTICAL MOVEMENT: Let $y(t)$ represent the vertical location of the tank at some time t, measured in minutes. Repeat questions (i-iv) for the vertical component, assuming $y_o = 4450$.

With the set of parametric equations that describe the tanks movement answer the following questions: What are the coordinates of the tank 2 minutes after it starts moving?

If the tanks horizontal location is 5000, what is it's vertical location?

Assuming the tank continues to move in the same direction and the same speed, where will it be 1 hour later (assume the final coordinates are on the same map sheet)?

3. Sketch the curve represented by the parametric equations

$$x = e^t,\ y = \sqrt{t},\ 0 \leq t \leq 1$$

(a) Indicate with an arrow the direction in which the curve is traced as t increases.

(b) Eliminate the parameter to find a Cartesian equation of the curve.

4. Describe the motion of a particle with position (x, y) as t varies in the given interval.

   (a) $x = \cos \pi t,\ y = \sin \pi t, \quad 1 \leq t \leq 2$

   (b) $x = 2 + \cos t,\ y = 3 + \sin t, \quad 0 \leq t \leq 2\pi$

   (c) $x = 2 \sin t,\ y = 3 \cos t, \quad 0 \leq t \leq 2\pi$

   (d) $x = \cos^2 t,\ y = \cos t, \quad 0 \leq t \leq 4\pi$