MA205 - Integral Calculus
Lesson 21: Motion in Space I

Mechanics Based Problems

1. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle for the interval $-1 \leq t \leq 2$ and draw the velocity and acceleration vectors for the specified value of t. Plot the particle's speed as a function of time.

$$\vec{r}(t) = \langle t^2 - 1,\ t \rangle,\ t = 1$$

2. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and initial position.

$$\vec{a}(t) = \mathbf{k}\ ,\ \vec{v}(0) = \mathbf{i} - \mathbf{j}\ ,\ \vec{r}(0) = \mathbf{0}$$

3. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and initial position.

$$\vec{a}(t) = -10\mathbf{k}\ ,\ \vec{v}(1) = \mathbf{i} + \mathbf{j} - \mathbf{k}\ ,\ \vec{r}(1) = \mathbf{i} + \mathbf{k}$$

Problem Solving Problems

1. A projectile is fired with an initial speed of $500m/s$ and an angle of elevation $30°$. Find the range of the projectile, the maximum height reached, and the speed at impact.

2. Rework the previous problem if the projectile is fired from a position $200m$ above the ground.

3. A ball is thrown at an angle of $45°$ to the ground. If the ball lands $90m$ away, what was the initial speed of the ball?

4. A gun has muzzle speed $150m/s$. Find two angles of elevation that can be used to hit a target $800m$ away.

5. A batter hits a baseball $3ft$ above the ground toward the center field fence, which is $10ft$ high and 400 ft from home pate. The ball leaves the bat with speed 115 ft/s at an angle of $50°$ above the horizontal. Is it a home run? (In other words does the ball clear the fence?)