

**MA205 - Integral Calculus**  
**Lesson 27: Estimations Using Data and Functions II**

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Mechanics Based Problems

1. Approximate the volume under the surface

$$f(x, y) = (x - 3y^2)$$

over the region  $R$  where  $0 \leq x \leq 2$  and  $1 \leq y \leq 2$  using  $m = n = 2$  choosing the sample point to be the (a) upper left corner, (b) upper right corner, (c) lower left corner, (d) lower right corner and (e) midpoint of each subrectangle. Calculate your first approximations by hand then use technology to compute the same values.



3. Estimate the volume of the space under the surface  $z = 50 - x^2 - y^2$  and above the surface  $z = x^2 + y^2$ .