# Lesson 54 - Systems of First-Order Differential Equations

Objectives

- Use multiple differential equations to model a multi-container mixing problem.
- Write a system of linear, first-order differential equations in a matrix form.
- Determine if solutions of a system of differential equations are linearly independent.
- Verify a solution to a system of differential equations.

READ

- Differential Equations Supplemental, section 8.1 (Preliminary Theory).
- Lesson 54 Course Guide Supplemental Reading.

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

**Finding the determinant of a matrix**:
The command `Det` can be used to find the determinant of a matrix. For example, to find the determinant of the matrix

$$\begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix}$$

You can input the following into Mathematica:

```
In[4]:= Det[(3 4
             1 0)]

Out[4]= -4
```

Lesson 54 Course Guide Supplemental Reading

We can develop a system of first order differential equations to model a multi-container mixing problem. Consider the mixing problems from earlier in this block.

Consider Problem Solving Problem #1 of Lesson 45:
*A tank contains 200 liters of a fluid in which 30 grams of salt is dissolved. Brine containing 1 gram per liter is then pumped into the tank at a rate of 4 L/min; the well mixed solution is pumped out at the same rate. Find the number $A(t)$ of grams of salt in the tank at $t = 10$ minutes and $t = 20$ minutes.*

What if we added another tank to this system? Say, for example, that the brine flowing out of the first tank (described in the problem) flowed into another tank, this one 100 liters, containing 10 grams of a salt. If the mixture is removed at a the same rate at which it was introduced, how can we model the amount of salt in both of these tanks?

The differential equation that models the rate of change of salt in the first tank remains the same:

$$\frac{dA}{dt} = 4 - \frac{4}{200}A$$

where $A(0) = 30$. Let's define a new variable $B(t) =$ The number of grams of salt into the second tank after t minutes. Now, the overall rate of change of the amount of salt in the second tank is a function of how much is coming in FROM the first tank. That is,

$$\frac{dB}{dt} = \frac{4}{200}A - \frac{4}{100}B$$

where $B(0) = 10$. Now we have the system of differential equations:

$$\begin{aligned}\frac{dA}{dt} &= 4 - \frac{4}{200}A \\ \frac{dB}{dt} &= \frac{4}{200}A - \frac{4}{100}B\end{aligned}$$

where $A(0) = 30$ and $B(0) = 10$.