MA205 - Integral Calculus and Intro to Differential Equations

PROJECT I - 150 points

Instructions: The following assignment is due on **24 OCT 08 IN CLASS**. In accordance with the Documentation of Written Work (DWW), you must document any and all collaboration with sources other than your partner, your textbook, and your instructor. Documentation must be IAW Paragraph 1 Documentation Standard of Section VI Complying with Documentation Standards from the DWW. Work will be assessed for your group performance. Evaluation and grading will account for the use of sources in addition to your textbook. Sufficient work is required to indicate clearly the method of reasoning and the operations performed. **SHOW ALL WORK**.

The Stark Enterprises (SE) CEO has decided to pursue new business opportunities and is considering building an amusement park. The design of the amusement park will focus around the main roller coaster, the Iron Monger. The engineers of SE have done a great deal of preliminary testing and calculations, but seem to be behind schedule. Therefore, SE is outsourcing some of the remaining tasks and would like your consulting company to perform some analysis on a few areas of the design. The engineers at SE have designed the shape of the roller coaster with the piecewise equation shown below. (They are also providing you with a Mathematica notebook which has these equations already coded up.)

$$RC(t) = \begin{cases}
\{-1000. + 2.\,t,\ -100.,\ 50.\} & 0. < t \le 7.5 \\
\{-985. + 10.\,\mathrm{Sin}[0.2\,(-7.5+t)],\ -100.,\ 50. + 10.\,(1. - 1.\,\mathrm{Cos}[0.2\,(-7.5+t)])\} & 7.5 < t \le 11.427 \\
\{-977.929 + 2.\,(-11.427+t),\ -100.,\ 52.9289 + 2.\,(-11.427+t)\} & 11.427 < t \le 151.427 \\
\{-697.929 + 10.\,(0.707107 - 1.\,\mathrm{Sin}[0.785398 + 0.2\,(151.427 - 1.\,t)]), & 151.427 < t \le 161.899 \\
\quad -100.,\ 332.929 + 10.\,(-0.707107 + \mathrm{Cos}[0.785398 + 0.2\,(151.427 - 1.\,t)])\} & \\
\{-843.098 + t,\ -100.,\ 68.4461 + 0.0133925\,(-301.233+t)^2\} & 161.899 < t \le 301.233 \\
\{-541.865 + 0.5\,(-301.233+t),\ -100.,\ 68.4461\} & 301.233 < t \le 501.233 \\
\{-441.865 + 600.\,(1. - 1.\,2.71828^{0.00666667\,(501.233-1.\,t)}), & 501.233 < t \le 901.233 \\
\quad -100. + 0.125\,(-501.233+t)\,\mathrm{Sin}[0.0314159\,(-501.233+t)], & \\
\quad 68.4461 + 0.125\,(-501.233+t)\,(-1. + \mathrm{Cos}[0.0314159\,(-501.233+t)])\} & \\
\{116.445 + 50.\,\mathrm{Sin}[0.020944\,(-901.233+t)], & 901.233 < t \le 976.233 \\
\quad -34.6719 + 100.\,\mathrm{Cos}[3.14159\,(1.125 + 0.01\,(-901.233+t))] & \\
\qquad \mathrm{Cos}[6.28319\,(1.125 + 0.01\,(-901.233+t))], & \\
\quad 95.5059 + 100.\,\mathrm{Cos}[6.28319\,(1.125 + 0.01\,(-901.233+t))] & \\
\qquad \mathrm{Sin}[3.14159\,(1.125 + 0.01\,(-901.233+t))]\} & \\
\{166.445 - 600.\,(-0.0183156 + 2.71828^{0.01\,(-1376.23+t)}), & 976.233 < t \le 1376.23 \\
\quad 30.6563 + 0.125\,(1376.23 - 1.\,t)\,\mathrm{Sin}[0.0314159\,(-976.233+t)], & \\
\quad 68.4461 + 0.125\,(1376.23 - 1.\,t)\,(-1. + \mathrm{Cos}[0.0314159\,(-976.233+t)])\} & \\
\{953.667 - 1.\,t,\ 30.6563,\ 68.4461\} & 1376.23 < t \le 1451.23 \\
\{-497.566 - 0.571429\,(-1451.23+t),\ 30.6563, & 1451.23 < t \le 2151.23 \\
\quad 68.4461 + 200.\,(0.0909457 + \mathrm{Sinc}[3.14159\,(3.5 + 0.005\,(1451.23 - 1.\,t))])\} & \\
\{-897.566 + 0.5\,(2151.23 - 1.\,t),\ 30.6563,\ 286.635\} & 2151.23 < t \le 2551.23 \\
\{-1097.57 + 65.\,\mathrm{Cos}[3.14159\,(0.5 + 0.01\,(-2551.23+t))], & 2551.23 < t \le 3051.23 \\
\quad 30.6563 + 65.\,(-1. + \mathrm{Sin}[3.14159\,(0.5 + 0.01\,(-2551.23+t))]), & \\
\quad 286.635 - 0.47\,(-2551.23+t)\} & \\
\{-1097.57 + 48.7829\,(-3051.23+t), & 3051.23 < t \le 3053.23 \\
\quad -99.3437 - 0.328148\,(-3051.23+t),\ 51.6352 - 0.817596\,(-3051.23+t)\} &
\end{cases}$$

Figure 1: Roller Coaster Equation

A 3D plot of the coaster and the surrounding terrain is shown below.

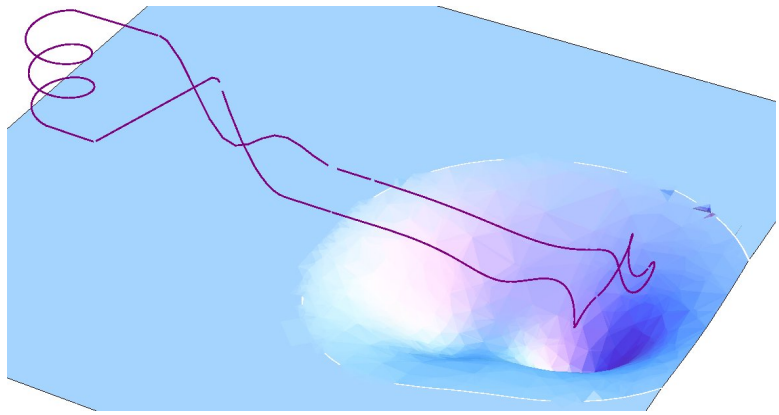

Figure 2: Roller Coaster Plot

The roller coaster equation, represented by the vector function $RC(t)$, determines the path of the track as a function of the independent variable t. The $z(t)$ component indicates the elevation (in feet) of the roller coaster above ground level. Although the support structure of the roller coaster is not going to be a solid support wall, research has shown that the cost of the support structure is well approximated by the vertical area between the 3D space curve and the ground. Given that the parametric vector function for the roller coaster is represented by

$$RC(t) = \langle x(t), y(t), z(t) \rangle \,,$$

the area under a space curve can be determined by the following integral:

$$\int_a^b z(t)\sqrt{x'(t)^2 + y'(t)^2}dt, \text{ where } \quad t \in [a, b]$$

SE would like to know the total cost of materials and labor to build the roller coaster's track and supporting structure given the following cost matrix (which include parts and labor):

	Track Materials	Support Materials
Cost in \$ / ft of track length	75	N/A
Cost in \$ / ft^2 of area under track	N/A	120

Table 1: Track Cost Matrix

The track materials are used to construct the rail portion of the roller coaster and support materials make up the support frame of the track. Table 1 already accounts for the track width with respect to the total cost.

To increase its novelty, portions of the roller coaster will be built over a small lake. The engineers at SE have determined that five concrete pylons must be built under the lake to support the track frame above the water. Each pylon will be rectangular in shape and extend from the water surface to the bottom of the lake bed. The size and location of each pylon (from an aerial view) is given in Table 2 and pictured in Figure 3, where the first coordinate corresponds to the southwest corner of the rectangle and the second coordinate corresponds to the northeast. Each pylon will cost an average of \$500 per cubic foot.

Pylon	LL-coord	UR-coord
1	(-300,10)	(-150,35)
2	(-300,-110)	(-150,-85)
3	(60,-135)	(100,-80)
4	(140,15)	(165,65)
5	(110,-75)	(200,0)

Table 2: Pylon Coordinates

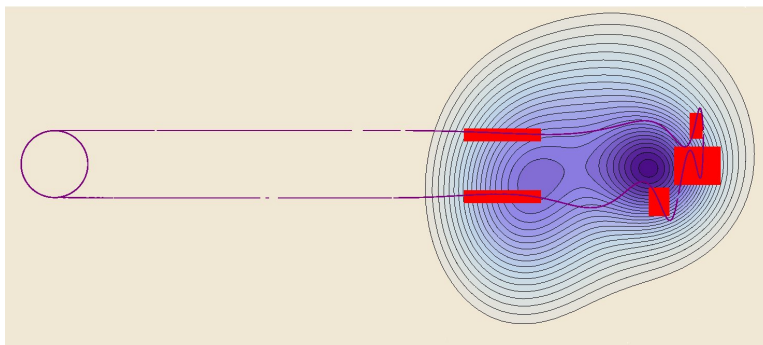

Figure 3: Aerial view of Track, Lake Bed contours, and Pylons

The lake bed is modeled by the following equation that measures the depth (in feet).

$$\text{LakeBed}(x,y) = \text{Min}[0, 10 - 150(e^{\frac{-(x^2+y^2)}{50{,}000}} + e^{\frac{-(2(x+200)^2+(y+100)^2)}{40{,}000}} + e^{\frac{-((x-75)^2+2(y+50)^2)}{10{,}000}})]$$

Finally, two small portions of the roller coaster will run under the water. The engineers at SE designed an underwater tunnel for the coaster. They conducted preliminary testing and calculations, but are still concerned about whether it can withstand the hydrostatic forces on the sides of the tunnel. The engineers need to know the total amount of hydrostatic force that each flat and curved face of the tunnel must withstand. The following diagram depicts the dimensions of the sidewall (See your Stewart textbook, *Calculus: Early Transcendentals*, Section 8.3, for a discussion on hydrostatic force). The tunnel is 10 feet wide and is curved on the sides following the formula $f(x,y) = \frac{x^2}{135} - 15$.

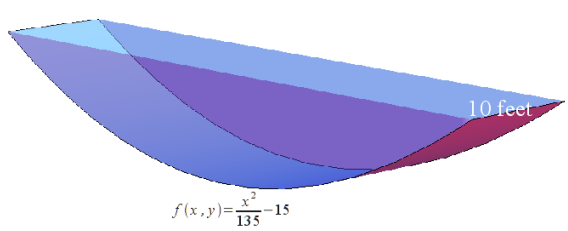

Figure 4: Roller Coaster Tunnel Sidewall Diagram

Your task is to estimate the total cost (track, support structure, concrete pylons) to build the *Iron Monger*, and to determine the underwater hydrostatic pressure on the faces of the tunnel. Provide a written report to the SE CEO outlining your methodology and final results. Be sure to state any relevant assumptions to the problem, show all supporting work, and justify why your results are sound.