Mechanics Based Problems

1. Approximate the volume under the surface

$$f(x, y) = (x - 3y^2)$$

over the region R where $0 \leq x \leq 2$ and $1 \leq y \leq 2$ using $m = n = 2$ choosing the sample point to be the (a) upper left corner, (b) upper right corner, (c) lower left corner, (d) lower right corner and (e) midpoint of each subrectangle. Calculate your first approximations by hand then use technology to compute the same values.

$= -8.75$ Lower Left

$= -17.75$ upper Left

$= -15.75$ upper right

$= -6.75$ lower right

$= -11.875$ midpoint

ans

Problem Solving Problems

1. Let V be the volume of the solid that lies under the graph of $f(x,y) = \sqrt{52 - x^2 - y^2}$ and above the rectangle given by $2 \le x \le 4$, $2 \le y \le 6$. We use the lines $x = 3$ and $y = 4$ to divide R into subrectangles. Let L and U be the Riemann sums computed using lower left corners and upper right corners, respectively. Without calculating the numbers V, L, and U, arrange them in increasing order and explain your reasoning.

$V \approx 47.42$ lower left

$V \approx 41.57$ upper right

2. Plot the solid $\sqrt{9 - y^2}$ on the region $R = [0,4] \times [0,2]$. Now estimate the volume of this solid with 2 subrectangles and 4 subrectangles and compare your results.

$V \approx 17.88$ upper right $m = 1, n = 2$

$V \approx 20.25$ upper right $m = n = 2$

3. Estimate the volume of the space under the surface $z = 50 - x^2 - y^2$ and above the surface $z = x^2 + y^2$.

$V \approx 2500$ cubes

upper circle using lower right endpts

lower " " upper " " "