Lesson #18 Answers

Mechanics Based Problems

1. Graph the space curve $\vec{r}(t) = \langle \sin(t), \cos(t), t^2 \rangle$ with Mathematica. Choose three different parameter intervals: one where t is always negative, one where t is always positive, and one where t is both positive and negative. You can add the following commands to your `ParametricPlot3D` command to make the plot look better: `BoxRatios→{1,1,1},PlotPoints→150,PlotRange→All`.

2. Evaluate the following integrals:

   (a) $\int_0^1 (16t^3\,\mathbf{i} - 9t^2\,\mathbf{j} + 25t^4\,\mathbf{k})dt$ (evaluate by hand)

   $$\underline{\underline{(4i - 3j + 5k)}}$$

   ANS

   (b) $\int_1^4 (\sqrt{t}\,\mathbf{i} + te^{-t}\,\mathbf{j} + \frac{1}{t^2}\,\mathbf{k})dt$ (use technology)

   $$\underline{\underline{\left(\frac{14}{3} i + .644 j + \frac{3}{4} k\right)}}$$

   ANS

   (c) $\int (e^t\,\mathbf{i} + 2t\,\mathbf{j} + 1/t\,\mathbf{k})dt$ (evaluate by hand)

   $$\underline{\underline{\left(e^t i + t^2 j + \ln|t|\, k\right) + \vec{C}}}$$

   ANS

(d) $\int(\cos \pi t\ \mathbf{i} + \sin \pi t\ \mathbf{j} + t\ \mathbf{k})dt$ (evaluate by hand and with technology)

$$\left(\frac{\sin(\pi t)}{\pi}\ i - \frac{\cos(\pi t)}{\pi}\ j + \frac{t^2}{2}\ k\right) + \vec{C}$$

ANS

Problem Solving Problems

1. Find $\vec{r}(t)$ if $\vec{r}\,'(t) = t^2\mathbf{i} + 4t^3\mathbf{j} - t^2\mathbf{k}$ and $\vec{r}\,(0) = \mathbf{j}$.

$$\vec{r}(t) = \frac{t^3}{3}\ i + (t^4+1)\ j - \frac{t^3}{3}\ k$$

ANS

2. Find $\vec{r}(t)$ if $\vec{r}\,'(t) = \sin t\ \mathbf{i} - \cos t\ \mathbf{j} + 2t\ \mathbf{k}$ and $\vec{r}(0) = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

$$\vec{r}(t) = (-\cos t + 2)\ i - (\sin t - 1)\ j + (t^2+2)\ k$$

ANS

3. A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$. Find its position function.

$\vec{r}(t) = (t^3 + t)\,i + (t^4 - t)\,j - (t^3 - 3t)\,k$

ANS