1. When we estimate distances from velocity data it is sometimes necessary to use times $t_0, t_1, t_2, t_3, \ldots$ that are not equally spaced. We can still estimate distances using the time periods $\Delta t_i = t_{i-1}$. For example, on May 7, 1992, the space shuttle Endeavor was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

Event	Time(s)	Velocity (ft/s)
Launch	0	0
Begin Roll Maneuver	10	185
End roll Maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

(a) Plot this data in excel or mathematica. Label your axes. Print out the graph.

(b) Sketch rectangles to approximate the area covered by the data.

(c) Estimate how high the shuttle is above the earth's surface after 62 seconds. Provide a lower and upper estimate of the shuttle's height.

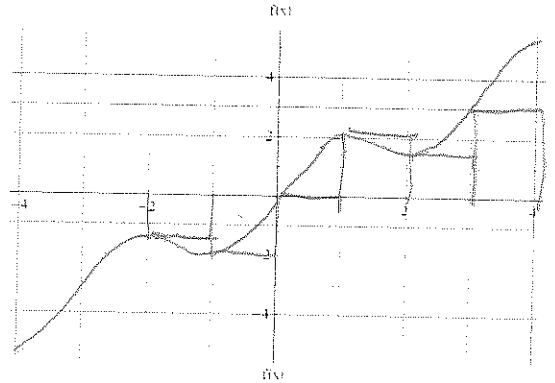
c) Lower Estimate = 31,893 ft ANS

Upper Estimate = 54,694 ft ANS

2. Use six rectangles and left endpoint, right endpoint, midpoint, and trapezoidal rules to find estimates of the area under the graph given below from $x = -2$ to $x = 4$. How do your answers compare? Which are over and which are underestimates? Sketch all 4 techniques on the graphs below.

LEFT

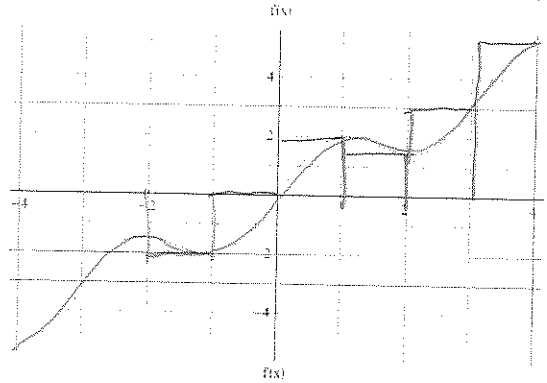
3 UNITS ANS



UNDER

RIGHT

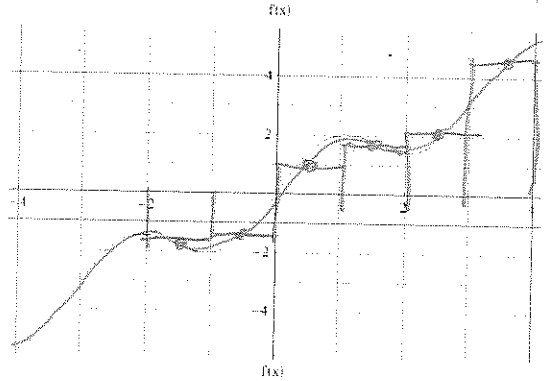
9 UNITS ANS



OVER

MIDPOINT

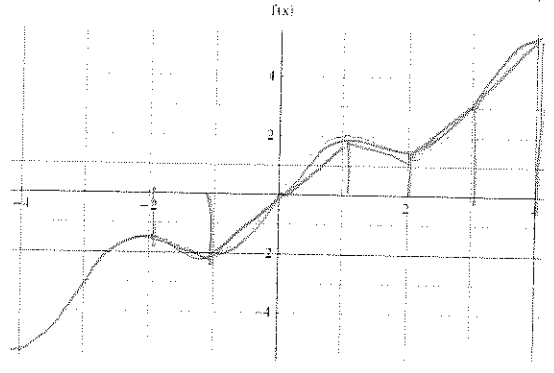
6.5 UNITS ANS



UNDER

TRAPEZOIDAL

6.625 UNITS ANS



UNDER

Problem Solving Problems

1. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table below. Find lower and upper estimates for the distance that she traveled during these three seconds.

time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
velocity (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

upper

44.8 ft

Ans

lower

34.7 ft

Ans

2. Racing cars driven by Chris and Kelly are side by side at the start of a race. The table below shows the velocities of each car in miles per hour during the first ten seconds of the race. Estimate how much farther Kelly travels than Chris does during the first ten seconds.

t	v_{Chris}	v_{Kelly}	t	v_{Chris}	v_{Kelly}
0	0	0	6	69	80
1	20	22	7	75	86
2	32	37	8	81	93
3	46	52	9	86	98
4	54	61	10	90	102
5	62	71			

127.6 ft farther

Ans

3. A friend of yours sent you the following data on the velocity of an Unladen European Swallow (its a rare bird) he observed. The negative velocities indicate traveling from "right to left" based on the location of the observer, while positive velocities indicate motion from "left to right".

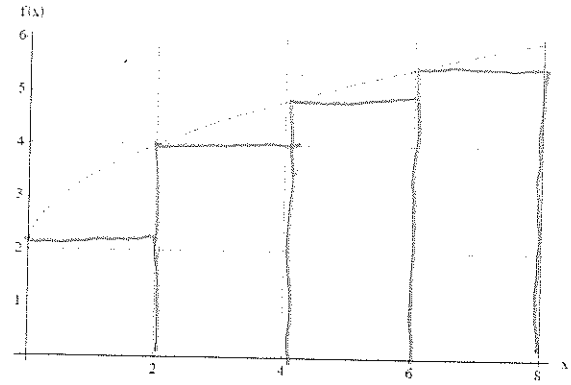
time(sec)	0	2	7	11	16	21	28	33	41
velocity(ft/sec)	20	-5	16	21	-3	8	13	-1	6

Can you tell him how far the Swallow flew? Where, approximately, in relation to the origin is the swallow after 41 seconds?

USING RIGHT ENDPOINT : 373 ft traveled ANS

BIRD ENDS 313 ft to the right of the ORIGIN. ANS

4. By reading values from the given graph of f, use four rectangles to estimate the area between the graph of f and the x axis from $x = 0$ to $x = 8$. Sketch the rectangles that you use on the graph. Is your estimate an underestimate or an overestimate? Justify your reasoning.



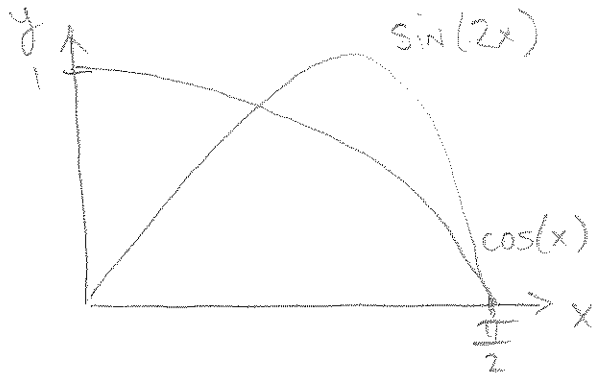
USING LEFT ENDPOINTS

31.6 UNITS

UNDER ESTIMATE - ALL ARE BELOW THE CURVE

ANS

5. Use Mathematica to plot the region that lies between the curves $y = \cos(x)$ and $y = \sin(2x)$ from $x = 0$ to $x = \pi/2$. Notice that the region consists of two separate parts. Estimate the area of this region using techniques you have learned.



USING RIGHT ENDPOINTS $\Rightarrow$ .412 UNITS

ANS