Mechanics Based Problems

1. Estimate the area under the curve $y = x$ from 1 to 2 using 10, 30, and 50 subintervals with all the techniques you have learned. What is the exact area? Record your estimates in the table below.

Technique	$n = 10$	$n = 30$	$n = 50$
Left	1.45	1.48333	1.49
Right	1.55	1.51667	1.51
Midpoint	1.5	1.5	1.5
Trapezoid	1.5	1.5	1.5

See following pages...

2. If $f(x) = \sin(\sin(x))$, where $0 \leq x \leq \pi/2$ use left, right, trapezoid, and midpoint rules with 10, 30, and 50 subintervals to estimate the area under the curve. Show that the exact area under f lies in between 0.87 and 0.91. Record your estimates in the table below.

Technique	$n = 10$	$n = 30$	$n = 50$
Left	.825097	.870986	.879944
Right	.957275	.915045	.906379
Midpoint	.894273	.893358	.893285
Trapezoid	.891186	.893015	.893161

See following pages...

Problem Solving Problems

1. You have seen the table below, provided by NASA, that gives the velocity data for the shuttle Endeavor between liftoff and the jettisoning of the solid rocket boosters. Fit a function to this data and use left, right, trapezoid, and midpoint rules with appropriate subintervals to estimate the height of the shuttle at 62 seconds. Compare your estimate with the estimate you obtained in lesson 2. Do the estimates make sense? Can you adjust your subintervals so the answers converge?

Event	Time(s)	Velocity (ft/s)
Launch	0	0
Begin Roll Maneuver	10	185
End roll Maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

In light of the work above, discuss the meaning of the expressions:

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ = Right Hand Technique

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\Delta x$ Left endpoint technique

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} f(\frac{x_{i-1}+x_i}{2})\Delta x$ Midpoint Approximation

(d) $\lim_{n\to\infty} \left(\frac{\sum_{i=1}^{n} f(x_i)\Delta x + \sum_{i=1}^{n} f(x_{i-1})\Delta x}{2}\right)$ = Trapezoidal Approximation

(See following pages for more work. .)

with the $\lim_{n\to\infty}$, each of these approximations approaches the true value of the area under the curve, since as n gets big, Δx becomes very small.