Mechanics Based Problems

For the following exercise use Euler's method with a step size of $h = 0.1$, *and again with* $h = 0.05$ *to obtain numerical solutions for the DE and stated initial condition. Perform these initial computations by hand and record your numerical solutions in a table. Then use Mathematica to obtain numerical solutions to the problems using step sizes of* $h = 0.001$ *and* $h = 0.0001$. Compare the different solutions.

1. $y' = 2x - 3y + 1, y(1) = 5$; find $y(1.2)$

$h = 0.01 \Rightarrow y(1.2) = 2.98$

$h = 0.05 \Rightarrow y(1.2) = 3.11514$

$h = 0.001 \Rightarrow y(1.2) = 3.22623$

$h = 0.0001 \Rightarrow y(1.2) = 3.22811$

Problem Solving Problems

For the following four exercises estimate a proportionality constant if needed, then use Euler's method to determine a numerical solution, pick an appropriate step size.

1. A tank contains 200 liters of fluid in which 30 grams of salt is dissolved. Brine containing 1 gram of salt per liter is then pumped into the tank at a rate of 4 L/min; the well mixed solution is pumped out at the same rate. Find the number $A(t)$ of grams of salt in the tank at $t = 10$ minutes and $t = 20$ minutes.

for $h = 0.1$

$A(10) = 60.84$ grams of salt

$A(20) = 86.09$ grams of salt

2. A large tank is filled to capacity with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well mixed solution is pumped out at 10 gal/min. How long does it take for the tank to be empty?

100 minutes

3. Select an appropriate step size for Problem Solving Problem #~~4~~ 5. Lesson 39, and perform the following:

   (a) Estimate the temperature of the cup of coffee after 10 minutes using Euler's Method. Compare your numerical results (from Euler's Analysis) to your graphical results. Do the results appear to be in agreement with each other?

for $h = 0.1$

$T(10) = 81.39\ ^{\circ}C$

   (b) According to your numerical analysis, how long does it take for the coffee to cool down to room temperature?

Approximately 250 minutes

4. A large tank contains 300 gallons of saltwater. A brine solution enters the tank at a rate of 3 gallons per minute. The concentration of the in flow is variable and is given by $c_{in}(t) = 2 + \sin(t/4)$ in pounds per gallon. The solution in the tank is pumped out at a rate of 3 gallons per minute. If 50 pounds of salt is initially dissolved in the tank, how much salt is in the tank after 10 minutes? After 2 hours? After 10 days? What is the long term behavior of salt in the tank?

for $h = 1$

$A(10) = 122.5$ lbs

$A(120) = 438.1$ lbs

$A(14{,}400) = 588.65$ lbs

Long term → oscillates between 300 & 900