46 $X \sim$ inside diameter of piston ring $\sim N(\mu_X = 12\text{ cm}, \sigma_X = 0.04\text{ cm})$

a. $\bar{X}$ is normally distributed.

$$\mu_{\bar{X}} = \mu_X = 12\text{ cm}$$

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{0.04}{\sqrt{16}} = \underline{0.01\text{ cm}}_{\text{ANS}}$$

b. $\mu_{\bar{X}} = \mu_X = 12\text{ cm}$

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{0.04}{\sqrt{64}} = \underline{0.005\text{ cm}}_{\text{ANS}}$$

c. The larger random sample ($n = 64$) has a $\sigma_{\bar{X}} = 0.005$. This means that 95% of all $\bar{X}$ will be within 0.01 cm from $\mu_{\bar{X}}$. The smaller sample size will only be within 0.01 cm of $\mu_{\bar{X}}$ 68.2% of the time.

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48 $X \sim$ wt of 50 lb bag of fertilizer

C.L. thm $\bar{X} \sim N\left(\mu_{\bar{X}} = \mu_X = 50,\ \sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{100}} = 0.1\right)$

a. $P(49.9 \le \bar{X} \le 50.1) = 0.682689$ (1 standard deviation)

b. $P(49.9 \le \bar{X} \le 50.1) = 0.157305$

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50 $X \sim$ breaking strength of a rivet

C.L. thm $\bar{X} \sim N\left(\mu_{\bar{X}} = \mu_X = 10000\text{ psi},\ \sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{500}{\sqrt{40}} = 79.05694\right)$

a. $P(9900 \le \bar{X} \le 10200) = \underline{0.891342}_{\text{ANS}}$

b. if $n = 15$, the sample may be too small to apply the Central Limit Theorem. If X is normal, then you could still apply C.L. thm.

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51 $X \to$ time to fill out a mortgage application

$X \sim N(\mu_X = 10\text{ min}, \sigma_X = 2\text{ min})$

"event"

A $\bar{X}_1 \sim N\left(\mu_{\bar{X}_1} = 10,\ \sigma_{\bar{X}_1} = \frac{\sigma_X}{\sqrt{5}} = \frac{2}{\sqrt{5}}\right) \to P(\bar{X}_1 \le 11) = \int_{-\infty}^{11} f(x)\,dx = 0.868224$

B $\bar{X}_2 \sim N\left(\mu_{\bar{X}_2} = 10,\ \sigma_{\bar{X}_2} = \frac{\sigma_X}{\sqrt{6}} = \frac{2}{\sqrt{6}}\right) \to P(\bar{X}_2 \le 11) = 0.889664$

$$P(A \cap B) = P(A)P(B) = \underline{0.772428}_{\text{ANS}}$$

53 $X \sim$ rockwell Hardness of pin

$X \sim N(\mu_X = 50, \sigma_X = 1.2)$

a. $P(\bar{X} \geq 51) = 1 - P(\bar{X} \leq 51)$

by C.L.Th. $\bar{X} \sim N(\mu_{\bar{X}} = \mu_X = 50, \sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{1.2}{\sqrt{9}})$

$$\int_{51}^{\infty} f(x)dx = \underline{0.00621}$$ Ans

b. $\bar{X} \sim N(\mu_{\bar{X}} = 50, \sigma_{\bar{X}} = \frac{1.2}{\sqrt{45}})$

$$\int_{51}^{\infty} f(x)dx = \underline{6.604 \times 10^{-8}}$$ Ans