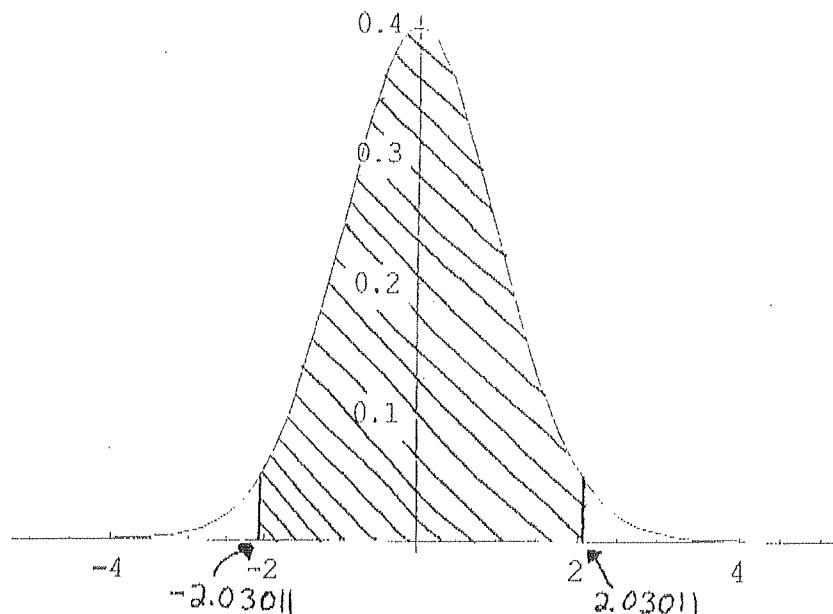


MA206 Suggested Problems
Student's t-Distribution

The following graph is the PDF of a t-distribution with $\nu = 35$ degrees of freedom.



- a. Estimate the middle 95% of all the possible values of T and shade the area under the curve for these values in the picture above. This area will be equal to 0.95. Should the remaining areas to the left and right of the shaded area be equal to each other? Why or why not?

The T-distribution is symmetric around $x=0$ (y-axis). The area to the right of 2.03011 is equal to the area to the left of -2.03011. (both are 0.025)

- b. At what value does the shaded region begin? At what value does the shaded region end? Are these values equal in magnitude? Why or why not?

$$-2.03011 \rightarrow 2.03011$$

Both endpoints (limits of integration) have the same absolute value because of the symmetry in this problem.

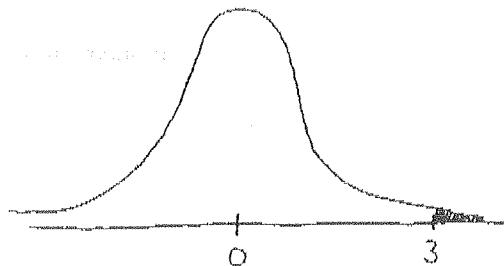
- c. If we used 85% instead of 95%, answer both a and b above.

We exploit the same symmetry as before.

Our new points are: $-1.47184 \rightarrow 1.47184$

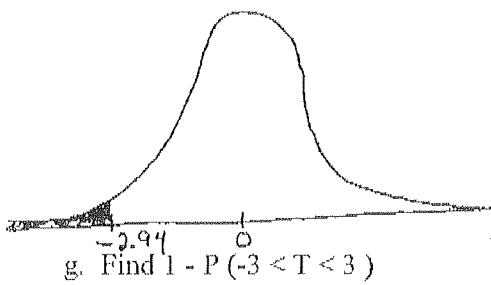
Let $f(x)$ be the PDF of the T-distribution
with $v = 35$ degrees of freedom.

d. Find the $P(T > 3.0)$



$$\int_3^{\infty} f(x) dx = 0.00247$$

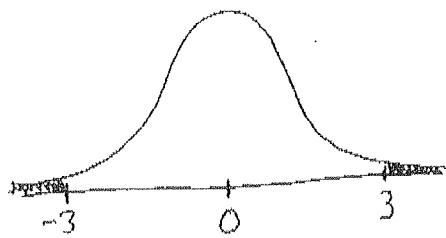
e. Find $P(T < -2.94)$



$$\int_{-\infty}^{-2.94} f(x) dx \approx 0.00289$$

g. Find $1 - P(-3 < T < 3)$

$$1 - P(-3 < T < 3)$$



$$\begin{aligned} & \int_{-\infty}^{-3} f(x) dx + \int_3^{\infty} f(x) dx \\ & \stackrel{\text{OC}}{=} \\ & 2 \cdot \int_3^{\infty} f(x) dx \Rightarrow (2)(0.00247) \\ & = 0.00494 \end{aligned}$$

from part d

h. In your own words, describe the probability that you calculated in part g above. (Translate the probability statement)

- $P(-3 < T < 3)$ is the probability that we are in the "middle" of the PDF \Rightarrow between -3 and 3
- $1 - P(-3 < T < 3)$ is the probability that we are in one of the two tails
- By Symmetry, this is simply $2 * \text{probability of one tail}$