LESSON 15 (matrix operations and Gaussian Elimination) EXAMPLE PROBLEMS

① GIVEN $A = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix}$, $\vec{a} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 & 0 & 8 \end{bmatrix}$, FIND $(ab)A$

$$\vec{a}\vec{b} = \underset{3\times 1}{\begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}} \underset{1\times 3}{\begin{bmatrix} 3 & 0 & 8 \end{bmatrix}} = \begin{bmatrix} 5(3) & 5(0) & 5(8) \\ 1(3) & 1(0) & 1(8) \\ 2(3) & 2(0) & 2(8) \end{bmatrix}$$

$$= \underset{3\times 3}{\begin{bmatrix} 15 & 0 & 40 \\ 3 & 0 & 8 \\ 6 & 0 & 16 \end{bmatrix}}$$

$$\Rightarrow (ab)A = \underset{3\times 3}{\begin{bmatrix} 15 & 0 & 40 \\ 3 & 0 & 8 \\ 6 & 0 & 16 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 15(6) + 0(10) + 40(-10) & 15(-2) + 0(-3) + 40(5) & 15(-2) + 0(1) + 40(1) \\ 3(6) + 0(10) + 8(-10) & 3(-2) + 0(-3) + 8(5) & 3(-2) + 0(1) + 8(1) \\ 6(6) + 0(10) + 16(-10) & 6(-2) + 0(-3) + 16(5) & 6(-2) + 0(1) + 16(1) \end{bmatrix}$$

$$= \underset{3\times 3}{\begin{bmatrix} -310 & 170 & 10 \\ -62 & 34 & 2 \\ -124 & 68 & 4 \end{bmatrix}}$$

Ans

② SOLVE VIA GAUSSIAN ELIMINATION AND BACK SUBSTITUTION

$$\begin{aligned} x_1 - x_2 + x_3 &= 0 \\ -x_1 + x_2 - x_3 &= 0 \\ 10x_2 + 25x_3 &= 90 \\ 20x_1 + 10x_2 &= 80 \end{aligned}$$

⇒ AUGMENTED MATRIX $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right]$

PIVOT 1 →
ELIMINATE THESE →

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right] \begin{array}{l} \\ \text{ROW } 2 + \text{ROW } 1 \\ \\ \text{ROW } 4 - 20 \text{ ROW } 1 \end{array} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{array}\right]$$

NOTE WE CAN'T PIVOT ON 0 TO ELIMINATE 10 AND 30
⇒ MOVE ROW 2 TO THE BOTTOM

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{ROW } 3 - 3 \text{ ROW } 2 \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & -95 & -190 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

PIVOT 10 TO ELIMINATE 30 (NOTE THAT 0 IN THE LAST ROW IS ALREADY "ELIMINATED")

WE NOW HAVE ROW ECHELON FORM IN OUR AUGMENTED MATRIX ⇒ USE BACK SUBSTITUTION TO SOLVE FOR x_1, x_2, x_3

$$\begin{aligned} \Rightarrow \quad x_1 - x_2 + x_3 &= 0 && \longrightarrow \quad x_1 - 4 + 2 = 0 \Rightarrow x_1 = 2 \\ 10x_2 + 25x_3 &= 90 && \longrightarrow \quad 10x_2 + 25(2) = 90 \Rightarrow x_2 = 4 \\ -95x_3 &= -190 && \longrightarrow \quad x_3 = 2 \\ 0 &= 0 \end{aligned}$$

↑ DON'T NEED

∴ OUR SOLUTION IS $\vec{x} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$

ANS