EXAMPLE PROBLEMS, MA 364 Lesson #2:

① Calculate $(1+i)^5$

$z = 1+i, \quad n = 5$

$r = |z| = \sqrt{x^2+y^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}(1) = \frac{\pi}{4}$

$\Rightarrow (1+i)^5 = \sqrt{2}^5\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right]$

$= 4\sqrt{2}\cdot\left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$

$= -4-4i$

ANS

NOTE:
$z^n = r^n(\cos n\theta + i\sin n\theta)$

② Find all the roots: $\sqrt[3]{3+4i}$

$z = 3+4i, \quad n = 3$

$r = |z| = \sqrt{x^2+y^2} = \sqrt{3^2+4^2} = 5$

$\theta = \tan^{-1}\frac{4}{3}$

$\Rightarrow \sqrt[3]{3+4i} = \sqrt[3]{5}\left(\cos\frac{\theta+2k\pi}{3} + i\sin\frac{\theta+2k\pi}{3}\right), \quad k = 0, 1, 2$

$\Rightarrow w_1 = \sqrt[3]{5}\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right)$

$w_2 = \sqrt[3]{5}\left(\cos\frac{\theta+2\pi}{3} + i\sin\frac{\theta+2\pi}{3}\right)$

$w_3 = \sqrt[3]{5}\left(\cos\frac{\theta+4\pi}{3} + i\sin\frac{\theta+4\pi}{3}\right)$

where $\theta = \tan^{-1}\frac{4}{3}$

ANS

NOTE: $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta+2k\pi}{n} + i\sin\frac{\theta+2k\pi}{n}\right), \quad k = 0, 1, \ldots, n-1$