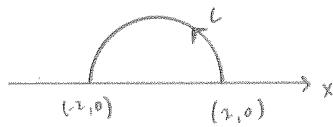


## EXAMPLE PROBLEMS, LESSON #24 (LINE INTEGRALS)

1. CALCULATE THE LINE INTEGRAL FOR  $\vec{F} = \langle x^2, y^2, 0 \rangle$ , C: SEMICIRCLE FROM  $(2,0)$  TO  $(-2,0)$ ,  $y \geq 0$



$$\text{PARAMETERIZATION: } \vec{r}(t) = \langle 2\cos t, 2\sin t \rangle$$

$$\begin{aligned} \text{NOTE THAT } \vec{r}(0) &= \langle 2, 0 \rangle \text{ AND } \vec{r}(\pi) = \langle -2, 0 \rangle \\ \Rightarrow t_0 &= 0, \quad t_f = \pi \end{aligned}$$

$$\vec{r}'(t) = \langle -2\sin t, 2\cos t \rangle$$

$$\vec{F}(\vec{r}) = \langle (2\cos t)^2, (2\sin t)^2, 0 \rangle = \langle 4\cos^2 t, 4\sin^2 t, 0 \rangle$$

$$\Rightarrow \int_{t_0}^{t_f} \vec{F}(\vec{r}) \cdot \vec{r}'(t) dt = \int_0^\pi \langle 4\cos^2 t, 4\sin^2 t, 0 \rangle \cdot \langle -2\sin t, 2\cos t, 0 \rangle dt$$

$$= \int_0^\pi (-8\cos^2 t \sin t + 8\sin^2 t \cos t) dt$$

$$= -8 \int_0^\pi \cos^2 t \sin t dt + 8 \int_0^\pi \sin^2 t \cos t dt$$

↓

$$u = \cos t, \quad du = -\sin t dt$$

$$8 \int u^2 du = \frac{8}{3} u^3$$

$$\begin{array}{l} \downarrow \\ u = \sin t, \quad du = \cos t dt \\ 8 \int u^2 du = \frac{8}{3} u^3 \end{array}$$

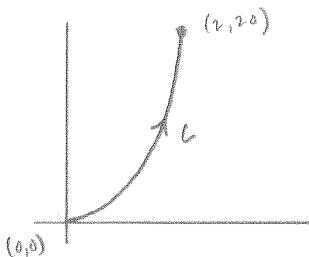
$$= \frac{8}{3} (\cos t)^3 \Big|_0^\pi + \frac{8}{3} (\sin t)^3 \Big|_0^\pi$$

$$= \frac{8}{3} (-1)^3 - (-1)^3 + \frac{8}{3} (0)^3$$

$$= \frac{8}{3} (-2) = -\frac{16}{3}$$

~~A ds~~

2. COMPUTE THE LINE INTEGRAL FOR  $\vec{F} = \langle y^3, x^3 \rangle$ , C: PARABOLA FROM  $(0,0)$  TO  $(2, 20)$



$$\text{PARAMETERIZATION: } \vec{r}(t) = \langle t, 5t^2 \rangle$$

NOTE THAT  $\vec{r}(0) = \langle 0, 0 \rangle$  AND  $\vec{r}(2) = \langle 2, 20 \rangle$   
 $\Rightarrow t_0 = 0, t_f = 2$

$$\vec{r}'(t) = \langle 1, 10t \rangle$$

$$\vec{F}(\vec{r}) = \langle (5t^2)^3, (t)^3 \rangle = \langle 125t^6, t^3 \rangle$$

$$\begin{aligned} \Rightarrow \int_{t_0}^{t_f} \vec{F}(\vec{r}) \cdot \vec{r}'(t) dt &= \int_0^2 \langle 125t^6, t^3 \rangle \cdot \langle 1, 10t \rangle dt \\ &= \int_0^2 (125t^6 + 10t^4) dt \\ &= \left( \frac{125}{7} t^7 + 2t^5 \right) \Big|_0^2 \\ &= \frac{125}{7} (2)^7 + 2(2)^5 = 2285.71 + 64 \\ &= 2349.71 \end{aligned}$$

AWS