LESSON 8 (LAPLACE TRANSFORMS) EXAMPLE PROBLEMS

1. (p. 226 #2) FIND THE LAPLACE TRANSFORM OF $(t^2-3)^2$

$$\mathcal{L}\{(t^2-3)^2\} = \mathcal{L}\{t^4 - 6t^2 + 9\}$$

$$= \mathcal{L}\{t^4\} - 6\mathcal{L}\{t^2\} + 9\mathcal{L}\{1\}$$

$$= \frac{4!}{s^5} - 6\left(\frac{2!}{s^3}\right) + 9\left(\frac{0!}{s}\right)$$

$$= \frac{24}{s^5} - \frac{12}{s^3} + \frac{9}{s}$$

ANS

2. (p. 226 #9) FIND THE LAPLACE TRANSFORM OF e^{3a-2bt}

$$\mathcal{L}\{e^{3a-2bt}\} = \mathcal{L}\{e^{3a}e^{-2bt}\}$$

$$= e^{3a}\mathcal{L}\{e^{-2bt}\}$$

$$= e^{3a}\left(\frac{1}{s+2b}\right) = \frac{e^{3a}}{s+2b}$$

ANS

3. (p. 226 #7) FIND THE LAPLACE TRANSFORM OF $\cos(\omega t + \theta)$

NOTE THAT ω AND θ ARE CONSTANTS

BY IDENTITY, $\cos(\omega t + \theta) = \cos\omega t\cos\theta - \sin\omega t\sin\theta$

$$\Rightarrow \mathcal{L}\{\cos(\omega t+\theta)\} = \mathcal{L}\{\cos\omega t\cos\theta - \sin\omega t\sin\theta\}$$

$$= \cos\theta\,\mathcal{L}\{\cos\omega t\} - \sin\theta\,\mathcal{L}\{\sin\omega t\}$$

$$= \cos\theta\left(\frac{s}{s^2+\omega^2}\right) - \sin\theta\left(\frac{\omega}{s^2+\omega^2}\right)$$

$$= \frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$$

ANS