MA 304 LESSON 9 EXAMPLE PROBLEMS

① FIND THE INVERSE LAPLACE TRANSFORM OF $F(s) = \frac{20}{(s-1)(s+4)}$

PARTIAL FRACTIONS: $\frac{20}{(s-1)(s+4)} = \frac{A}{s-1} + \frac{B}{s+4}$

$\boxed{A}$ MULTIPLY THROUGH BY $(s-1) \Rightarrow \frac{20}{s+4} = A + \frac{B(s-1)}{s+4}$

LET $s = 1 \Rightarrow \frac{20}{1+4} = A + \cancel{\frac{B(1-1)}{1+4}}$ $\to 0$

$\frac{20}{5} = A \Rightarrow A = 4$

$\boxed{B}$ MULTIPLY THROUGH BY $(s+4) \Rightarrow \frac{20}{s-1} = \frac{A(s+4)}{s-1} + B$

LET $s = -4 \Rightarrow \frac{20}{-4-1} = \cancel{\frac{A(-4+4)}{-4-1}} + B$ $\to 0$

$\frac{20}{-5} = B \Rightarrow B = -4$

$\Rightarrow \mathcal{L}^{-1}\left\{\frac{20}{(s-1)(s+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{4}{s-1} - \frac{4}{s+4}\right\} = 4\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\}$

$= 4e^{t} - 4e^{-4t}$

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② FIND THE INVERSE LAPLACE TRANSFORM OF $F(s) = \frac{3}{s^4} + \frac{s-5}{(s-5)^2+9} + \frac{15}{s^2+4s+29}$

$\mathcal{L}^{-1}\left\{\frac{3}{s^4} + \frac{s-5}{(s-5)^2+9} + \frac{15}{s^2+4s+29}\right\} = \mathcal{L}^{-1}\left\{\frac{3}{s^4}\right\} + \mathcal{L}^{-1}\left\{\frac{s-5}{(s-5)^2+9}\right\} + \mathcal{L}^{-1}\left\{\frac{15}{s^2+4s+29}\right\}$

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{6}{s^4}\right\} = \frac{t^3}{2}$

$= e^{5t}\cos 3t$

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CONSIDER $\mathcal{L}^{-1}\left\{\frac{15}{s^2+4s+29}\right\}$

NOTE THAT, BY COMPLETING THE SQUARE, $\frac{15}{s^2+4s+29} = \frac{15}{(s+2)^2+25} = 3\left(\frac{5}{(s+2)^2+25}\right)$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{15}{s^2+4s+29}\right\} = 3\mathcal{L}^{-1}\left\{\frac{5}{(s+2)^2+25}\right\}$$

$$= 3e^{-2t}\sin 5t$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{3}{s^4} + \frac{s-5}{(s-5)^2+9} + \frac{15}{s^2+4s+29}\right\} = \frac{t^3}{2} + e^{5t}\cos 3t + 3e^{-2t}\sin 5t$$

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(3) FIND THE INVERSE LAPLACE TRANSFORM OF $\frac{4s-2}{s^2-6s+18}$

WE CAN'T FACTOR, SO LET'S COMPLETE THE SQUARE IN THE DENOMINATOR $\rightarrow$

$$\frac{4s-2}{s^2-6s+18} = \frac{4s-2}{(s-3)^2+9}$$

THIS DOESN'T LOOK QUITE LIKE ANY OF THE TRANSFORMS IN TABLE 6.1, BUT IT SHARES A DENOMINATOR FORM w/ #11 AND #12

WHICH IS THE BETTER CANDIDATE? #11 BECAUSE OF THE "s" TERM IN OUR NUMERATOR, BUT WE NEED TO INTRODUCE AN $(s-3)$ TERM $\Rightarrow$ $4s-2 = 4(s-3)+10$

$$\Rightarrow \frac{4s-2}{(s-3)^2+9} = \frac{4(s-3)+10}{(s-3)^2+9}$$

WHAT NOW? IF WE DO A TERM-BY-TERM DIVISION IN THE NUMERATOR, THEN IT IS CLEAR WE ARE "CLOSE" TO A SUM OF #11 AND #12 FROM TABLE 6.1 $\Rightarrow$ $\frac{4(s-3)+10}{(s-3)^2+9} = 4\left(\frac{s-3}{(s-3)^2+9}\right) + \frac{10}{(s-3)^2+9}$

THE ONLY MODIFICATION WE NEED NOW IS TO PUT OUT A FACTOR OF $\frac{10}{3}$ IN THE SECOND TERM TO MATCH UP w/ #12's FORM EXACTLY

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{4s-2}{s^2-6s+18}\right\} = \mathcal{L}^{-1}\left\{4\left(\frac{s-3}{(s-3)^2+9}\right) + \frac{10}{3}\left(\frac{3}{(s-3)^2+9}\right)\right\}$$

$$= 4\mathcal{L}^{-1}\left\{\frac{s-3}{(s-3)^2+9}\right\} + \frac{10}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s-3)^2+9}\right\}$$

$$= 4e^{3t}\cos 3t + \frac{10}{3}e^{3t}\sin 3t$$

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