Lesson #6 - EXTENSION TO p.90 #3 (RELATED TO QUESTION #6 on PS 2b)

CONSIDER $y'' + y' + 4.25y = 221 \cos 4.5t$

WE HAVE SHOWN THAT $y_{ss} = y_p = -12.8 \cos 4.5t + 3.6 \sin 4.5t$

LET'S TAKE THIS PROBLEM AND LOOK AT THE RESPONSE FOR A GENERAL ω

$\Rightarrow y'' + y' + 4.25y = F_0 \cos \omega t$ $\quad m=1,\ c=1,\ k=4.25,\ \omega_0 = \sqrt{4.25}$

$F_0,\ \omega$

WHAT CHANGES? LET'S FIND THE AMPLIFICATION C/F_0 AS A FUNCTION OF ω, WHERE C IS THE AMPLITUDE IN OUR REPRESENTATION

$$y_p = a \cos \omega t + b \sin \omega t$$
$$= \underline{\underline{C}} \cos(\omega t - \eta)$$

PHASE ANGLE

FROM EQN (14), p.88, § 2.8, WE HAVE $C = \sqrt{a^2+b^2} = \dfrac{F_0}{\sqrt{m^2(\omega_0^2-\omega^2)^2 + \omega^2 c^2}}$

WHERE $a = F_0 \dfrac{m(\omega_0^2-\omega^2)}{m^2(\omega_0^2-\omega^2)^2 + \omega^2 c^2}$

$b = F_0 \dfrac{\omega c}{m^2(\omega_0^2-\omega^2)^2 + \omega^2 c^2}$

SO, $\underset{\text{AMPLIFICATION FUNCTION}}{A(\omega)} = \dfrac{C}{F_0} = \dfrac{1}{\sqrt{m^2(\omega_0^2-\omega^2)^2 + \omega^2 c^2}}$

IN OUR EXAMPLE, $A(\omega) = \dfrac{1}{\sqrt{(1)^2(4.25-\omega^2)^2 + \omega^2(1)^2}} = \dfrac{1}{\sqrt{(4.25-\omega^2)^2 + \omega^2}}$

LET'S LOOK @ $A(\omega)$ AND IT'S MAXIMUM

DETAILS IN BOOK... 1ST AND 2ND DERIVATIVE TESTS $\rightarrow$ $\omega^2_{max} = \omega_0^2 - \dfrac{c^2}{2m^2}$

FREQUENCY WHERE $A(\omega)$ IS MAXIMUM

$\Rightarrow \omega^2_{max} = 4.25 - \dfrac{(1)^2}{2(1)^2} = \dfrac{15}{4}$

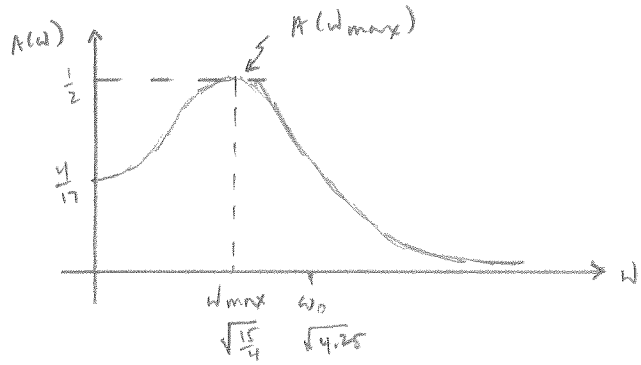
NEXT PAGE $\longrightarrow$

TOPS 35500

OBSERVE THAT AS c (DAMPING COEFFICIENT) $\to 0$, $\omega_{max} \to \omega_0$

$$A(\omega_{max}) = \frac{1}{\sqrt{(4.25-\omega_{max}^2)^2+\omega_{max}^2}} = \frac{1}{\sqrt{(4.25-\frac{15}{4})^2+\frac{15}{4}}} = \frac{1}{2}$$

GENERAL RESPONSE:



NOTE THAT AS OUR DAMPING COEFFICIENT c CHANGES, WE WILL GET DIFFERENT CURVES

WHAT IF OUR FORCING FUNCTION WERE $F_0 \sin \omega t$?

$$y_p = a \cos \omega t + b \sin \omega t$$
$$= C \cos(\omega t - \eta)$$

USING THE METHOD OF UNDETERMINED COEFFICIENTS, IT CAN BE SHOWN THAT

$$a = F_0 \frac{\omega c}{m^2(\omega_0^2-\omega^2)^2+\omega^2 c^2}$$

$$b = -F_0 \frac{m(\omega_0^2-\omega^2)}{m^2(\omega_0^2-\omega^2)^2+\omega^2 c^2}$$

$$\Rightarrow \quad C = \sqrt{a^2+b^2} = \frac{F_0}{\sqrt{m^2(\omega_0^2-\omega^2)^2+\omega^2 c^2}}$$

THIS IS THE SAME AS WHAT WE DERIVED FOR $F_0 \cos \omega t$!

SO, OUR ANALYSIS PROCEEDS IN AN IDENTICAL FASHION ...