Section 11.3 Even & Odd Functions
Half Range Expansions

Fourier Series of Even Function:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right)$$

Odd Function:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)$$

Graphical Example:

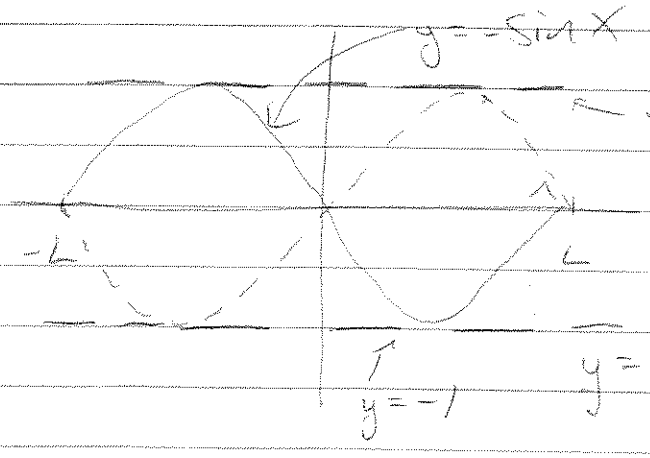
$y = -1$ ← even function, $f(-x) = f(x)$

↳ Graphically demonstrate that the b_n coefficients $= 0$

Step 1: $$b_n = \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

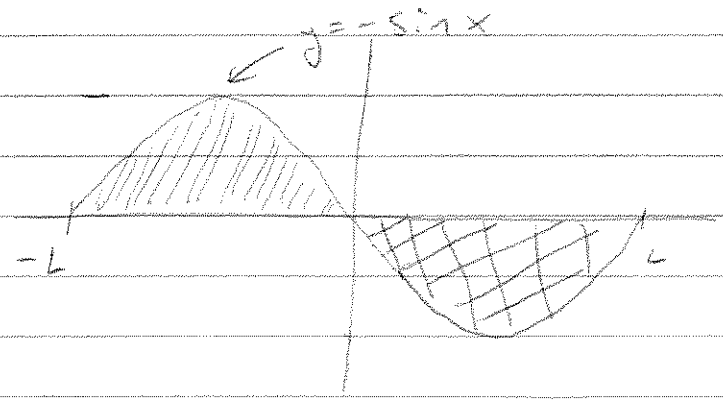
$$= \int_{-L}^{L} 1 \sin\left(\frac{n\pi x}{L}\right) dx$$

Step 2: Plot $f(x)$, $\sin(x)$ & $f(x) \cdot \sin(x)$



$y = \sin x$, ~~$y = 1 \cdot \sin x$~~

Step 3: What is The Integral?
Area Under Curve

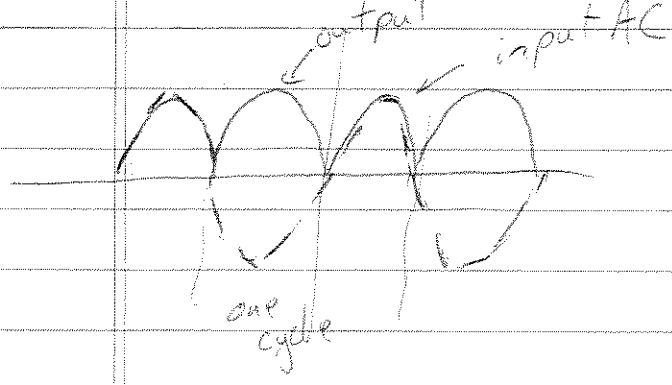


→ + area
→ - area

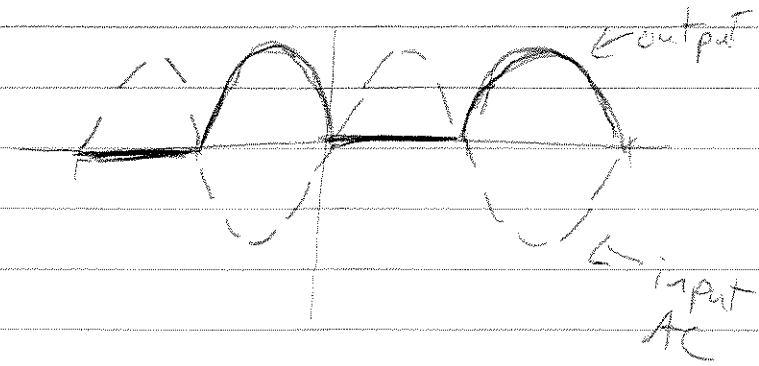
- area = + area, so
net sum of area = 0

→ Two Electrical Examples

Full Wave Rectifier
converts AC→DC rectifying
every half Cycle

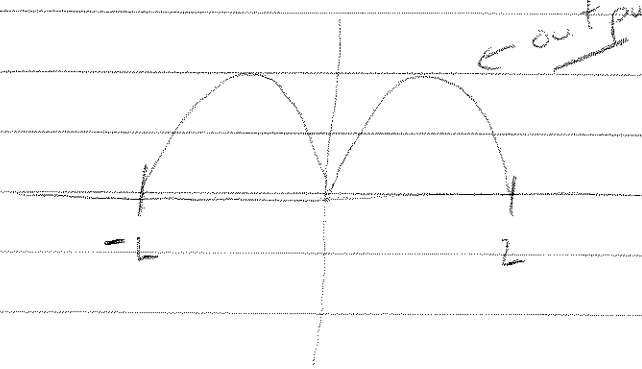


Half Wave Rectifier
converts AC→DC rectifying
every other half cycle &
blocking The other



Find the Fourier Series for Both:

a. Full-Wave: (Ideal)



input = $10 \sin 378t$

What is output function?

$f(x) = |10 \sin 378t|$

$$f(x) = \begin{cases} -10 \sin 378t & ? \le t \le ? \\ 10 \sin 378t & ? \le t \le ? \end{cases}$$

$\omega = 2\pi f$, $f = 60\,Hz$

$= 60$ Cycles/Second

T (period) $= .01667$

so

$$f(x) = \begin{cases} -10 \sin 378t & -.00833 \le t \le 0 \\ 10 \sin 378t & 0 \le t \le .00833 \end{cases}$$

Now that we have the function, we determine if even, odd, or neither.

In this case Even, so we solve for a_0, a_n, but not b_n

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi x}{L}\right) dx$$

Start with a_0:

Since F is split we have two integrals:

$$a_0 = \frac{1}{2L}\left(\int_{-L}^{0} -10\sin(2\cdot\pi\cdot 60\cdot t)\,dt + \int_{0}^{L} 10\cdot\sin(2\cdot\pi\cdot 60\cdot t)\,dt\right)$$

$$= \frac{1}{1/60}\cdot\left(\frac{10\cos(2\cdot\pi\cdot 60\cdot t)}{2\cdot\pi\cdot 60}\Bigg|_{-\frac{1}{120}}^{0} + \frac{-10\cos(2\pi 60 t)}{2\pi 60}\Bigg|_{0}^{1/120}\right)$$

$$= 60\cdot\left(\frac{10}{120\pi}\right)\cdot\left(\underbrace{\cos(0)}_{1} - \underbrace{\cos(-\pi)}_{-(-1)} + \underbrace{(-1)(\cos(\pi) - \cos(0))}_{-1((-1)-1)}\right)$$

$$= \frac{5}{\pi}\cdot(4)$$

$$= \frac{20}{\pi}$$

Next Find a_n:

$$a_n = \frac{1}{L}\left(\int_{-L}^{0} -10\sin(2\pi 60t)\cos\left(\cancel{n\pi 120t}\ \frac{n\pi t}{L}\right)dt + \int_{0}^{L} 10\sin(2\pi 60t)\cos\left(\frac{n\pi t}{L}\right)dt\right)$$

$$= \frac{1}{1/120}\left(\underbrace{\int_{-L}^{0} -10\sin(120\pi t)\cos(n120\pi t)\,dt}_{\text{u-sub ??}} + \underbrace{\int_{0}^{L} 10\sin(120\pi t)\cos(n120\pi t)\,dt}_{\text{u-sub ??}}\right)$$

No!

we have

$\sin(120\pi t)$ & $\cos(120n\pi t)$

Different ω's, so

need to use Trig Ids!

or MMA:

$$a_n = \frac{\cancel{120}\,240\,(1+(-1)^n)}{12\pi - 12n^2\pi}$$

? what happens when $n=1$?

$$a_1 = \frac{0}{0} \; ??$$

how do we solve?

Lets try $a_1 = 0, 1$ by plotting:

So:

$$f(x) = \frac{20}{\pi} + \begin{matrix} 1 \\ \text{or} \\ 0 \\ \text{or} \\ ? \end{matrix} + \sum_{n=2}^{K} \frac{240\,(1+(-1)^n)}{12\pi - 12n^2\pi} \cos(120n\pi x) \checkmark$$

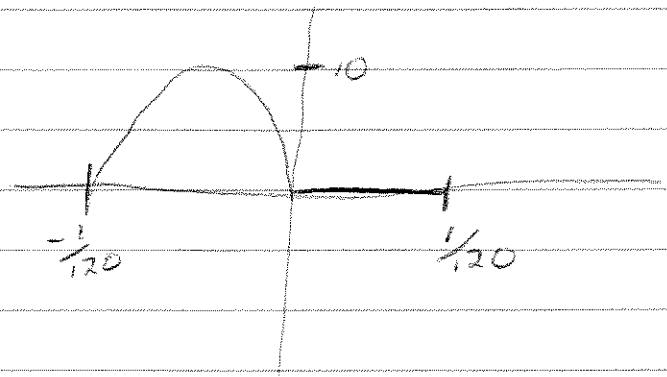
Plot in MMA:

$a_1 = ?$

$a_0 = 20/\pi$ ← DC component

Half-Wave Rectifier:
(Class Problem)

Step 1: Determine Your function based on:



Step 2: Determine if function is even or odd or neither

Step 3: Solve For The Coefficients:

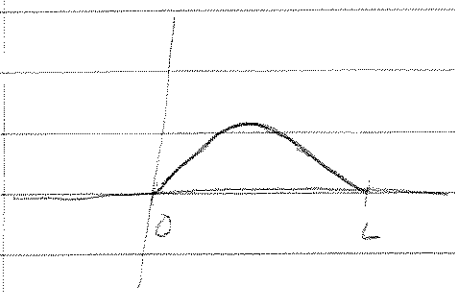
Step 4: Put together & Plot

Half Wave Expansions:

→ Useful in solving Differential Equations,

~~Imagine shape of violin~~

Imagine $f(x)$ representing the shape of a distorted violin string before release



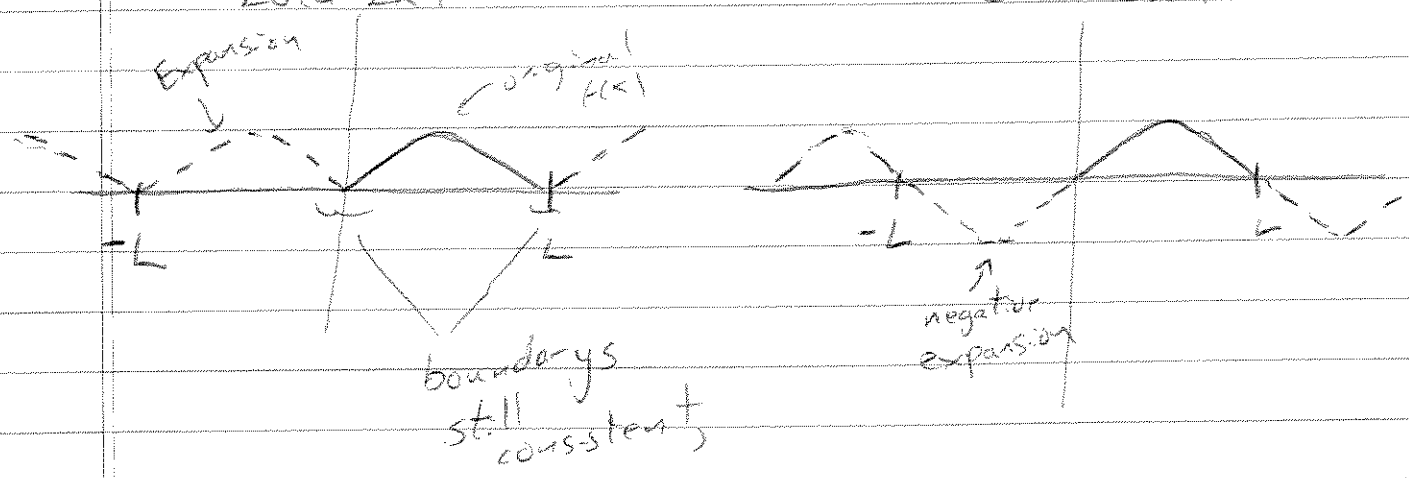
→ We could represent this as a Fourier Series & solve the governing differential Equations, but this contains both sines and cosines

→ Wouldn't it be better to only have Sines or Cosines??

When Solving The Diff Eq, we don't care about The solution outside our boundaries as long as The boundary conditions

So, We introduce Half-Range Expansions

Even Ext | Odd Ext



f(x) now only composed of cosines

f(x) now only composed of sine

Makes Solving Diff Eqs Easier