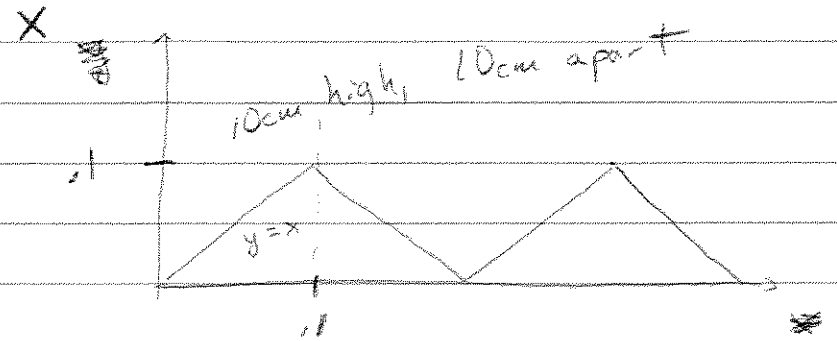
1.5 Model of Road ($x(t)$ will be road)



First: Parameterize road in terms of t, this takes into account the velocity we are traveling at:

Lets assume

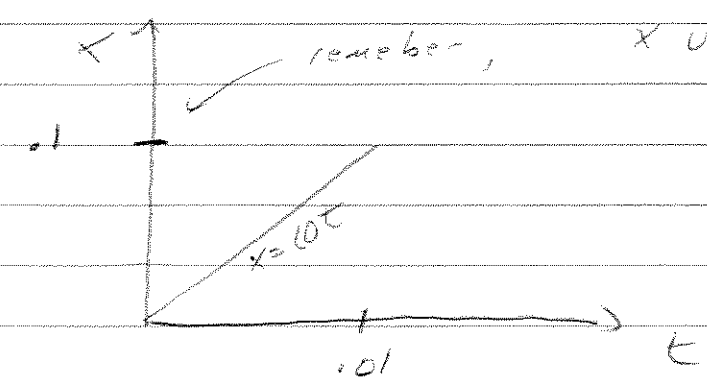
$x = vt = y$

where v = velocity

lets Assume $v = 10$ m/s

so $x(t) = 10t = y(t)$

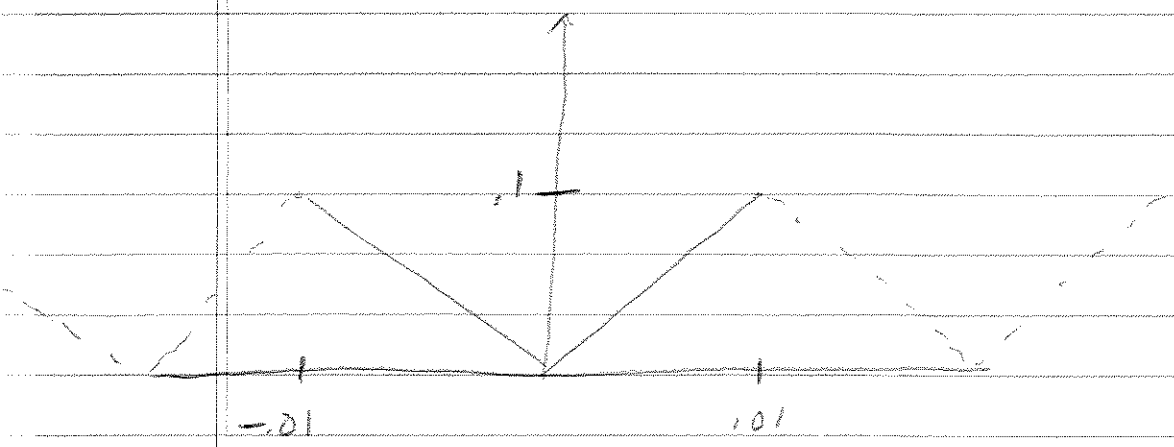
At this point, the car only feels the change in x, so plot x vs t



x valid between $0 \to .1$

if $x = .1$, $t = .01$

Since $x(t)$ will be periodic, do the even expansion of $x = 10t$, for $0 \le t \le .01$:



Find Fourier Coefficients:

a_n, b_n

Since even, $b_n = 0$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} x(t)\,dt = \text{avg value of } x(t)$$
$$= .05$$

$$a_n = \frac{1}{L}\int_{-L}^{L} x(t)\cos\left(\frac{n\pi t}{L}\right)dt$$

$$= \frac{1}{.01}\cdot 2\cdot\int_0^{.01} 10t\cdot\cos\left(\frac{n\pi t}{.01}\right)dt$$

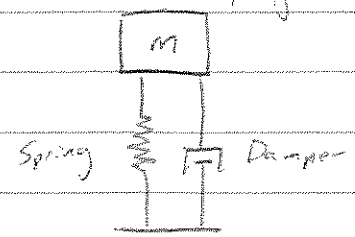
$$= 2000\cdot\int_0^{.01} \underbrace{10t\cos(100n\pi t)}_{\text{integration by parts}}\,dt$$

$$= \frac{2000\left(\cos(n\pi)-1\right)}{10000\,n^2\pi^2}$$

$\rightarrow$ higher velocity changes this # higher

Sol plotted in MATLAB

$y(t) =$ ~~with~~ displacement of vehicle



$$F_s = k(x - y)$$
$$F_D = \beta(x' - y')$$

$x(t) =$ road surface

$$m \cdot a = \sum F_s$$

$$m y'' = k(x - y) + \beta(x' - y')$$

$$\underbrace{m y'' + \beta y' + k y = k x + \beta x'}_{\text{simple model for a car suspension}}$$

Let $x(t) = \cos(\omega t)$ ⇐ solve for A & B
$x'(t) = \omega \sin(\omega t)$ in terms of ω

$$y(t) = A\cos(\omega t) + B\sin(\omega t)$$
$$y'(t) = -A\omega \sin(\omega t) + B\omega \cos(\omega t)$$
$$y''(t) = -A\omega^2 \cos(\omega t) + -B\omega^2 \sin(\omega t)$$

put x, x', y, y', y'' into model & Solve for A & B

I'll use MMA:

$$A = \frac{k^2 + \beta^2\omega^2 - km\omega^2}{k^2 + \beta^2\omega^2 - 2km\omega^2 + m^2\omega^4}$$
$$B = \frac{\beta m \omega^3}{k^2 + \beta^2\omega^2 - 2km\omega^2 + m^2\omega^4}$$

Convert $y(t)$ into $D(\omega)\cos(\omega t - \delta)$

$$D(\omega) = \sqrt{A^2 + B^2}$$
$$= \sqrt{\frac{K^2 + \beta^2\omega^2}{K^2 + \beta^2\omega^2 - 2Km\omega^2 + m^2\omega^4}}$$

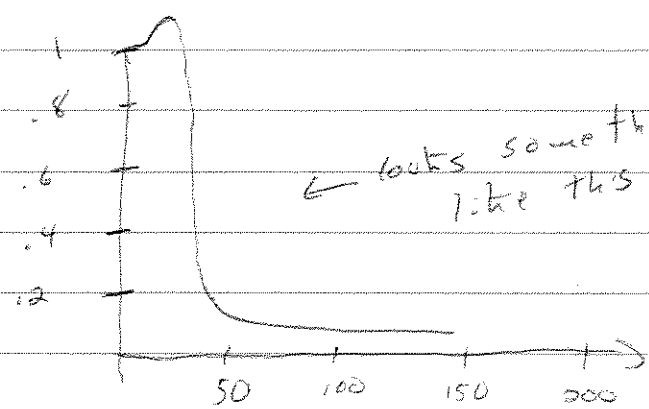
$$\delta = \text{Arctan}\left(\frac{B}{A}\right) = \text{Arctan}\left(\frac{\beta m\omega^3}{K^2 + \beta^2\omega^2 - Km\omega^2}\right)$$

Lets choose some values for
K, β, & m

$m = 300\,kg$
$K = 26260\ N/m$ → if my conversion is correct!
$\beta = ??$ 50 ← just a try
↑ 5613.55 after some research

Lets Plot $D(\omega)$ veses ω:

(view MMA file)



↑ So our shock system allows low frequencies through while damping out higher frequencies

Now using these values and our road for an input lets see what our ride is like.
(view MMA file)