Problem Set 1
Complex Numbers and Ordinary Differential Equations
MA 364, AY 09-2
Due Beginning of Lesson 8

1. (14 points) Express the following in rectangular form $(a + bi)$:

   (a) $\dfrac{1+2i}{3-4i} + \dfrac{2-i}{5i}$

   (b) $\left(\dfrac{1}{2-3i}\right)\left(\dfrac{1}{1+i}\right)$

2. (16 points) Convert the following to polar form:

   (a) $\dfrac{5+i}{26}$

   (b) $(1-i)^4$ convert to Cartesian notation also.

3. (16 points) Classify the following ordinary differential equations by order, linearity, homogeneity, and type of coefficients.

(a) $t^2 \dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin t$

(b) $(1 + y^2)\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$

(c) $\dfrac{dy}{dt} + ty^2 = 0$

(d) $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$

4. (14 points) Verify by substitution that the given functions form a basis. Solve the given initial value problem.

$$y_1(t) = t^{1/2}, \quad y_2(t) = t^{-1}, \quad 2t^2y'' + 3ty' - y = 0 \quad t > 0, \quad y(1) = 0.5, \quad y'(1) = 1$$

5. (15 points) Evaluate the following:

(a) Find a solution for:

$$2y'' + 3y' + y = 0$$

(b) Find the general solution for:

$$2y'' + 3y' + y = e^{-t/20} + 3\sin t$$

(c) Find a solution for the initial value problem:

$$2y'' + 3y' + y = e^{-t/20} + 3\sin t, \qquad y(0) = \frac{109}{90}, \quad y'(0) = \frac{13}{90}$$

6. (15 points)

(a) Present and ODE (Draw a free body diagram along with the differential equation!) which models the response $y(t)$ of a 15-kg mass to an external force $\sin(\omega t)$, a spring-like restoring force $3y(t)$, and a damping force of cy'. The mass begins from rest at the origin. (ω and c are both constants.)

(b) Let $c = 0$ giving us an undamped model then let $c = 1, 2, 3$. In each case, graph the maximum response (i.e., the maximum displacement fo the mass) as a function of ω as ω varies from 0 to 1. Plot all four curves together in a single plot.

(c) Let $c = 0, 1, 2, 3$. In each case, find the value of ω (accurate to 4 significant digits!) which maximizes the response.