**Problem Set 5**
**MA364, AY09-1**
**Differential Vector Calculus**

**Due Beginning of Lesson 24 (27 October)**

1. (10 pts) Pg. 398, #10

2. (10 pts) Pg. 399, #24

3. (10 pts) a. Pg. 409, #18.
   (5 pts) b. For the same problem, sketch some isotherms (level curves of $T$; curves on which $T =$ constant) and plot the gradient vector on them at the specified point.

4. (10 pts) Pg. 409, #30

5. (10 pts) Pg. 409, #34

6. (10 pts) Pg. 413, #2

7. (10 pts) Pg. 416, #4

PS#5 Solution

p. 398, #10

#1 Find a parametric representation for the helix $x^2+y^2=9$, $z=4\arctan(y/x)$

let $\left.\begin{array}{l} x = 3\cos t \\ y = 3\sin t \end{array}\right\} \Rightarrow x^2+y^2=9$

$\Rightarrow \quad y/x = \tan t \quad \Rightarrow$

$z = 4\arctan(\tan(t)) = 4t$

$\therefore \quad \underline{\underline{\vec{r}(t) = \langle 3\cos t,\ 3\sin t,\ 4t \rangle}}$ is a vector representation of the helix $x^2+y^2=9$, $z=4\arctan(y/x)$.

$\therefore$ parametric representations:

$$x(t) = 3\cos t$$
$$y(t) = 3\sin t$$
$$z(t) = 4t$$

ans

#2 p. 399, #24

Given $\vec{r}(t) = \langle 3\cos t, 3\sin t, 4t \rangle$, $P: (3, 0, 8\pi)$

Find a tangent vector, unit tangent, and the tangent of C at P. Sketch.

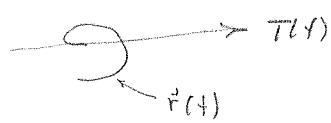
Note: $\vec{r}(2\pi) = \langle 3, 0, 8\pi \rangle$, which passes through $P(3, 0, 8\pi)$

$\underline{\underline{\vec{r}\,'(t) = \langle -3\sin t, 3\cos t, 4 \rangle}}$ is a tangent vector

$\dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$ = unit tangent vector = $\underline{\underline{\left\langle \frac{-3}{5}\sin t, \frac{3}{5}\cos t, 4/5 \right\rangle}}$

since $|\vec{r}\,'(t)| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + 4^2} = 5$

$\therefore$ tangent is given by $\underline{\underline{T(t) = \langle 3, 0, 8\pi \rangle + t\left\langle 0, \frac{3}{5}, \frac{4}{5} \right\rangle}}$ Ans



#3 (a) p.409, #18

Find $\vec{\nabla} T$ in general and at given P:

$T(x,y) = \sin x \cosh y \qquad P(\frac{\pi}{4}, \ln 5)$

$\vec{\nabla} T = \langle \cos x \cosh y, \sin x \sinh y \rangle$

$\vec{\nabla} T(P) = \langle \cos\frac{\pi}{4} \cosh(\ln 5), \sin\frac{\pi}{4} \sinh(\ln 5) \rangle$

$= \langle \left(\frac{\sqrt{2}}{2} \quad \frac{e^{\ln 5} + e^{-\ln 5}}{2}\right), \left(\frac{\sqrt{2}}{2} * \frac{e^{\ln 5} - e^{-\ln 5}}{2}\right) \rangle$ $\quad \frac{5.2}{2} = 2.6$

$\langle \frac{(5.2)\sqrt{2}}{4}, \frac{4.8\sqrt{2}}{4} \rangle$

So the direction of max decrease $= -\vec{\nabla} T(P) = \underline{\underline{\langle -1.3\sqrt{2}, -1.2\sqrt{2} \rangle}}$ Ans

(b) Sketch isotherms of $T(x,y) = \sin x \cosh y$

See attached Mathematica file

Mathematica command for level curves:

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ContourPlot[Sin[x]*Cosh[y],{x,-2Pi, 2Pi},{y,Exp[2],Exp[2]}]
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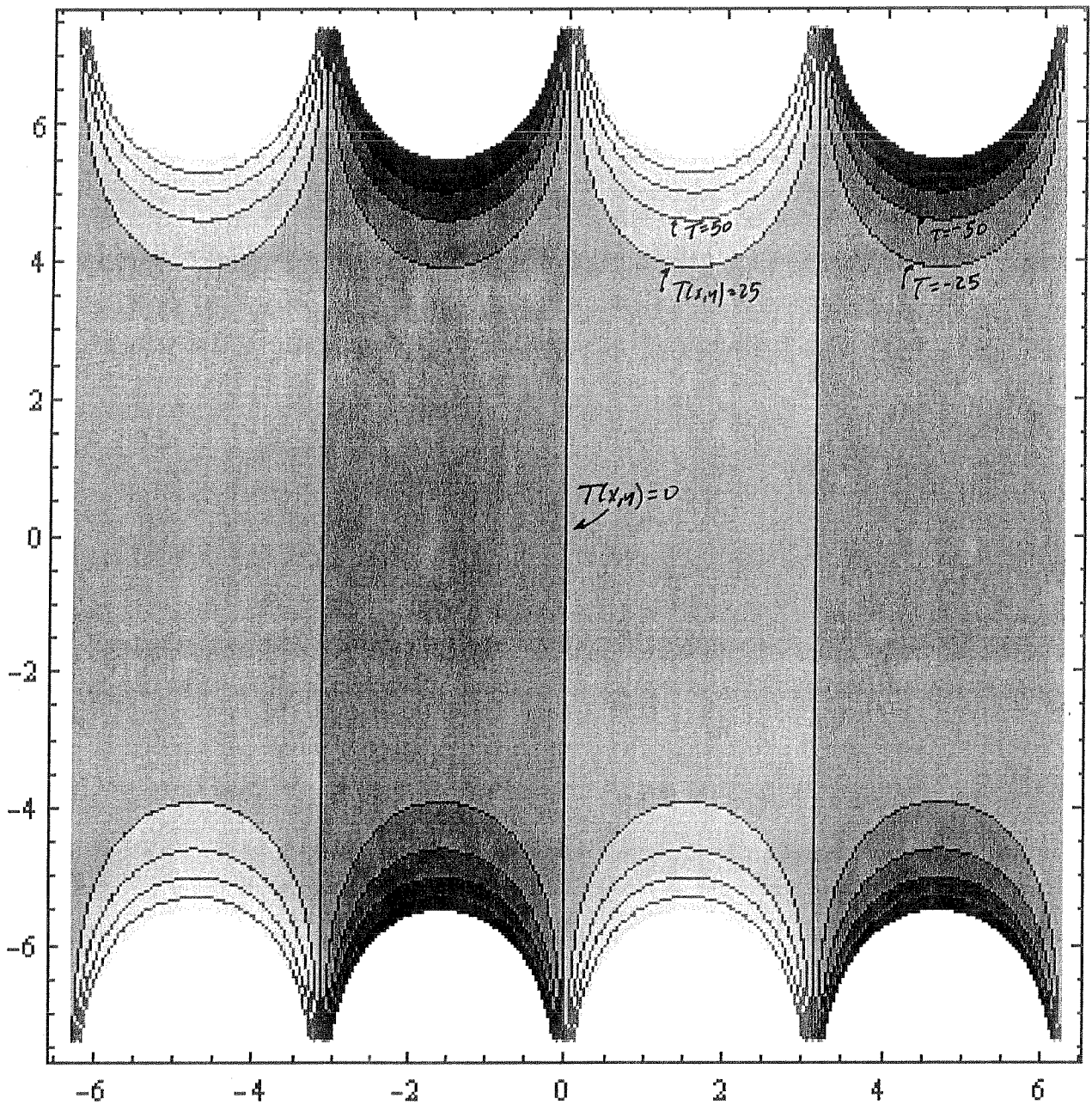

#4 p.409, #30

Find a normal vector to the surface at P

$x^2-y^2+4z^2=67$ , $P(-2,1,4)$ $\Rightarrow$ consider $f(x,y,z) = x^2-y^2+4z^2-67 = 0$

$\vec{n} = \vec{\nabla} f = \langle 2x, -2y, 8z \rangle$ $\perp$ to level surface $f=0$

so $\underline{\underline{\vec{n}(P) = \langle -4, -2, 32 \rangle}}$ Ans

#5 p.409, #34

Find the directional derivative of $f$ at $P$ in the direction of $\vec{a}$.

$f = x^2+y^2+z^2$ , $P(2,-2,1)$ $\vec{a} = \langle -1,-1,0 \rangle$

$\vec{\nabla} f(P) = \langle 2x, 2y, 2z \rangle \Big|_P = \langle 4, -4, 2 \rangle$

$\frac{\vec{a}}{|\vec{a}|} = \hat{u}_a = \langle -1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle$

$\therefore D_{\hat{a}} f = \langle 4,-4,2 \rangle \cdot \langle -1/\sqrt{2}, -1/\sqrt{2}, 0 \rangle$

$= \underline{\underline{0}}$ ans

#6 p.413, #2

Find the divergence of $\vec{F} = \langle e^{2x}\cos 2y,\ e^{2x}\sin 2y,\ 5e^{2z} \rangle$

$$\operatorname{div} \vec{F} = \vec{\nabla} \circ \vec{F}$$

$$= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \circ \left\langle e^{2x}\cos 2y,\ e^{2x}\sin 2y,\ 5e^{2z} \right\rangle$$

$$= 2e^{2x}\cos 2y + 2e^{2x}\cos 2y + 10e^{2z}$$

$$= \underline{\underline{4e^{2x}\cos 2y + 10e^{2z}}} \quad \text{Ans}$$

#7 p.416, #4

fn $\vec{v} = (x^2+y^2+z^2)^{-3/2} \langle x, y, z \rangle$ find curl $\vec{v}$

$$\text{curl } \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{x}{(\ )^{3/2}} & \frac{y}{(\ )^{3/2}} & \frac{z}{(\ )^{3/2}} \end{vmatrix} = \hat{i}\left(\frac{\partial}{\partial y}\frac{z}{(\ )^{3/2}} - \frac{\partial}{\partial z}\frac{y}{(\ )^{3/2}}\right) - \hat{j}\left(\frac{\partial}{\partial x}\frac{z}{(\ )^{3/2}} - \frac{\partial}{\partial z}\frac{x}{(\ )^{3/2}}\right)$$

$$+\hat{k}\left(\frac{\partial\, y/(\ )^{3/2}}{\partial x} - \frac{\partial}{\partial y}\, x/(\ )^{3/2}\right)$$

$$= \vec{0}$$

note $\vec{v} = \vec{\nabla} f$ where $f(x,y,z) = (x^2+y^2+z^2)^{1/2}$ and the

$\text{curl } \vec{\nabla} f = \vec{0}$ !