MA 371 Homework 1 Hints and guidance

For general guidelines, see "Homework Policies" link on the"Homework" page

1. I'll do part (a) as an example of both how to do the rest of the problem, and the type of argument I'd like to see.

   If $\mathbf{w} = (w_1, w_2)$ is a vector which is orthogonal to $\mathbf{v}$ then, by definition of orthogonal vectors, $\mathbf{v} \cdot \mathbf{w} = 0$. Thus

   $$1w_1 + 2w_2 = 0.$$

   Clearly, there are an infinite number of non-trivial solutions to the above equation. Solving for w_1 in terms of w_2 we obtain

   $$w_1 = -2w_2,$$

   which gives

   $$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = w_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

   So

   $$\mathbf{w} = t \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

   is orthogonal to $\mathbf{v}$ for any real number t.

   For parts (b)-(d), again use the fact that two vectors are orthogonal if their inner product is zero to obtain a system of equations. (For part (a), the system had just one equation.) We only have one technique for solving systems, so use it. You will have to use concepts like reduced row echelon form, and basic and free variables. (In part (a) the one basic variable was w_1 and the one free variable was w_2.)

2. Part (a) should remind you of the triangle inequality. When thinking about whether the statement is true or not, look at figure 1.2.10 on p.24, which depicts the Triangle Inequality. Think about how the picture would look if one vector is a scalar multiple of the other. Substitute $\mathbf{x} = k\mathbf{y}$ into the left-hand side of the equation and use property (c) of Theorem 1.2.2 to simplify. Then substitute $\mathbf{x} = k\mathbf{y}$ into the *right*-hand side of the equation and use the same property to simplify. Are they equal? If not, are they equal for *some* values of k?

   Part (b): again substitute $\mathbf{x} = k\mathbf{y}$ into both the right and the left-hand sides of the equation and simplify. You will need to use the property (c) of Theorem 1.2.6, as well as property (c) of Theorem 1.2.2

3. Part (a): Expand the left-hand side in terms of the inner product. What must occur for the result to be equal to the right-hand side?

   Part (b): Two sentences. I'll do this one for you:

   The statement is true, since if $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$ then

$$\begin{aligned} \mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) &= \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} \\ &= 0 + 0 \\ &= 0, \end{aligned}$$

   where we have used property (b) of Theorem 1.2.6. Therefore $\mathbf{u}$ is orthogonal to $\mathbf{v} + \mathbf{w}$.

   Part (c): A picture and one sentence will suffice here. For example, "The statement is ...., since if $\mathbf{u}$ and $\mathbf{v}$ are as pictured in the diagram, then ...."

4. Again, draw pictures.

5. Part (a): I discussed Sec. 2.2 Problem 47 in class. Also, there is a Mathematica notebook on the web which deals with the problem. Each equation represents a plane in $\mathcal{R}^3$. For each system, try graphing the planes.

   Part (b): The coefficient matrix for the first system is $\begin{pmatrix} 2 & 3 & -1 \\ 4 & 6 & -2 \end{pmatrix}$. The matrix comprises the following column vectors in $\mathcal{R}^2$: $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. The left-hand side of the system can be written as a linear combination of these vectors:

$$x \begin{pmatrix} 2 \\ 4 \end{pmatrix} + y \begin{pmatrix} 3 \\ 6 \end{pmatrix} + z \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

   How are the vectors related? What must occur for a linear combination to equal the vector on the right-hand side?

6. Part (a): You need not show any work here. If you find the RREF via TI-89 or Mathematica, you might state "Using Mathematica, we find the reduced row echelon form of A..."

   Part (b): You should take a few steps to write down the solution, using RREF from part (a).

   Part (c): You will have to find RREF of the augmented matrix here.

7. You're on your own here, except to say that for part (b), you want to use the RREF of the *augmented* matrix $A|\mathbf{b}$.