MA 371 Homework 2. Due the week of 10/13/03

1. Simplify: $(A^{-1}B^TC)^{-1}[CB^{-1}(A^{-1})^T]^T$

2. Sec. 3.2, #33.

3. Find conditions on k under which $A = \begin{pmatrix} k & 1 & 0 & 1 \\ 1 & k & 1 & 0 \\ 0 & 1 & k & 1 \\ 1 & 0 & 1 & k \end{pmatrix}$ is invertible, and find A^{-1}.

4. Let $A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & 0 \\ b & a & 0 & 0 & 0 \\ c & b & a & 0 & 0 \\ d & c & b & a & 0 \end{pmatrix}$. Find the smallest positive integer n such that A^n=0 by computing $A^2, A^3, A^4, \ldots$. (A matrix for which such an n exists is said to be *nilpotent.*)

5. For each of the following, find the 4×4 elementary matrix E such that the product EA has the given effect on the $4 \times n$ matrix A.

   (a) Interchanges rows 2 and 4.

   (b) Multiplies row 3 by -5 and adds it to row 2.

6. Use your answer to the above to find the 4 x 4 matrix B such that the product BA has the effect on A of interchanging rows 2 and 4, then subtracting 5 times row 3 from row 2.

7. Section 3.4 # 20. If the vectors *are* dependent, express v_1 as a linear combination of the other vectors.

8. Section 3.4 # 26.

9. Find the orthogonal complement in $\mathcal{R}^4$ of the vectors $v_1 = (1, -1, 0, -2), v_2 = (-1, 2, 1, 1)$ in $\mathcal{R}^4$.

10. Section 3.5 # 20.