MA 371 9/30/03.

1. Let $A = \begin{pmatrix} 2 & 1 & 3 & -2 \\ 4 & 2 & 1 & -2 \\ 6 & 3 & 4 & -4 \\ 4 & 2 & 1 & -2 \end{pmatrix}$ and $b = (7, 1, 8, 1)$.

   (a) Solve $Ax = 0$

   (b) Find two linearly independent vectors with integer entries which are solutions to $Ax = 0$, and show that their sum is also a solution.

   (c) Solve $Ax = b$.

   (d) Find two linearly independent vectors with integer entries (i.e., no fractions or decimals) which are solutions to $Ax = b$, and show that their sum is not a solution.

   (e) Find a basis for the row space of A.

(f) Find a basis for the column space of A.

2. Show that the vectors are linearly independent, or show that they are linearly dependent. If the latter is true, express v_3 as a linear combination of v_1 and v_2.

(a) $v_1 = (1, -2, 0), v_2 = (3, -1, 1), v_3 = (1, 2, 3)$.

(b) $v_1 = (0, -2, 1), v_2 = (1, 3, 1), v_3 = (1, 1, 2)$.

(c) $v_1 = (1, -1, 0, -2), v_2 = (-1, 2, 1, 1), v_3 = (1, 1, 2, 4)$.

3. Find the orthogonal complement in $\mathcal{R}^4$ of the vectors $v_1 = (1, -1, 0, -2), v_2 = (-1, 2, 1, 1)$ in $\mathcal{R}^4$.

4. Find the determinant of the following matrices.

$$A = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 1 & 0 & -2 \\ 0 & 2 & 0 & 0 \\ 3 & 3 & 4 & -4 \\ 0 & 2 & 0 & -2 \end{pmatrix}$$