MA 371 10/29/03. DDS, DE, Eigenvalues, vectors

1. A country is divided into three demographic regions. It is found that each year, 5% of the residents of Region 1 move to Region 2, and 5% move to region 3. Of the residents of Region 2, 15% move to Region 1 and 10% move to region 3. And of the residents of Region 3, 10% move to Region 1 and 5% move to region 2. Initially, 40% of the population resides in Region 1, 40% in Region 2, and 20% in Region 3. Find the percentage of the population residing in each region after k years. What percentage of the population resides in each region after a long time?

2. Solve system of differential equations.

$$\begin{aligned} x_1' &= 4x_1 + x_3 \\ x_2' &= -2x_1 + x_2 \\ x_3' &= -2x_1 + x_3 \end{aligned}$$

3. For the matrix in Prob. 8.(a), Find the eigenvalues and eigenvectors. Confirm that $PA = P\Lambda$, where P is the matrix $P = [v_1, v_2, v_3]$ whose columns are the eigenvectors, v_1, v_2, v_3, of A, and Λ is the diagonal matrix of eigenvectors.

4. What are the eigenvectors of a 4×4 diagonal matrix?