MA 371 9/12/03. Linear Equations, Matrix Operations

1. Write each of the following systems of equations in the form $A\mathbf{x} = \mathbf{b}$, and find the solution.

a)
$$\begin{aligned} u + 2v - 2w &= 0 \\ -u + w &= 0 \\ -2u + 2v + w &= 0 \end{aligned}$$

b)
$$\begin{aligned} 3x_1 + 3x_2 + 3x_3 &= -3 \\ -x_1 - 5x_2 + -2x_3 &= 3 \\ -4x_1 + 2x_3 &= 0 \end{aligned}$$

c)
$$\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ -1x_2 - 1x_3 + 2x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 0 \end{aligned}$$

d)
$$\begin{aligned} x_2 + x_4 &= 1 \\ x_1 - 2x_3 &= 1 \\ x_1 - x_2 + 2x_4 &= 1 \\ -x_1 + x_2 - x_3 - 2x_4 &= 1 \end{aligned}$$

2. Let $A = \begin{pmatrix} 2 & 1 & 3 & -2 \\ 4 & 2 & 1 & -2 \\ 6 & 3 & 4 & -4 \\ 4 & 2 & 1 & -2 \end{pmatrix}$, and $\mathbf{b} = (2, 1, 8, -4)^T$,

   (a) Consider the system $A\mathbf{x} = \mathbf{0}$. Identify the free and basic variables; find the solution. Is $\mathbf{x} = (0, 4, 2, 5)^T$ a solution? How does this agree with the solution you found?

   (b) Does $A\mathbf{x} = \mathbf{b}$ have a solution?

3. Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 3 & -1 \end{pmatrix}$, $F = \begin{pmatrix} -1 & 2 \\ 1 & 2 \end{pmatrix}$, $G = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, and $\mathbf{d} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Find

   (a) $A^T F$

   (b) $F^T A$

   (c) $(AB)^T_{2,1}$ Try doing this with as few multiplications as possible.

(d) $A\mathbf{b} \cdot \mathbf{d}$

(e) $\mathbf{b} \cdot A^T\mathbf{d}$

(f) $\mathbf{c}\mathbf{d}^T$

(g) $\mathbf{d}\mathbf{c}^T$

(h) F^{-1}.

4. Suppose $(5D^T)^{-1} = G$, with G given in the preceding problem. Find D.

5. Set up a system of equations whose solution is the set of coefficients of a cubic polynomial through the points $(-1, 1), (1, 3), (2, -1)$, and $(4, 10)$.