

Name: _____

Instructions

1. Use of notes and text is not authorized.
2. The use of the TI-89 is authorized, but not required.
3. Please use pencil.
4. Show all necessary work in the space provided on the exam itself; paper is provided for scratch work. This will not be graded.
5. All problems are equally weighted.

For problems 1 and 2, let $\mathbf{u} = (1, 2, 3, 4)$ and $\mathbf{v} = (-1, 3, 0, 1)$.

1. (a) Find $\mathbf{u} \cdot \mathbf{v}$.

(b) Find $3\mathbf{u} - 5\mathbf{v}$.

2. (a) Find a unit vector parallel to \mathbf{u} . (Recall that unit vectors have norm equal 1.)

(b) Find the Euclidean distance between \mathbf{u} and \mathbf{v} .

3. Suppose \mathbf{u} and \mathbf{v} are orthogonal vectors with $\|\mathbf{u}\| = 2$ and $\|\mathbf{v}\| = 3$. Find $\|5\mathbf{u}-2\mathbf{v}\|$.

For problems 4 and 5 let $A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & 2 \\ 3 & -1 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

4. (a) Find AB .

(b) Find $A^T\mathbf{b}$.

5. Determine whether $\mathbf{x} = (-12, 5, 2)^T$ is a solution to $A\mathbf{x} = \mathbf{b}$. Explain your reasoning.

6. Suppose wish to find a non-zero vector in \mathcal{R}^4 orthogonal to $\mathbf{v}_1 = [1, 2, 3, 0]$, $\mathbf{v}_2 = [-1, 0, 1, 1]$, and $\mathbf{v}_3 = [0, 2, 0, 1]$. Set up a matrix equation whose solution is the desired vector. **Do not solve.**

Problems 7 and 8 refer to the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 4 \end{pmatrix}$.

7. Show the steps in finding the reduced row echelon form of the matrix A .

8. Use your answer from Problem 7 to determine whether the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$ lie in the same plane. (**Hint:** Notice that the vectors are the columns of the matrix A . Recall that if three vectors lie in the same plane, then any one of the vectors can be expressed as a linear combination of the other two. What is the augmented matrix for the problem?)

9. Suppose that the reduced row echelon form of A is $\begin{pmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

(a) Identify the basic and the free variables for the equation $A\mathbf{x} = \mathbf{0}$.

(b) Write down the solution to $A\mathbf{x} = \mathbf{0}$.