MA 371 WPR 3 9 December 2003

Name:

1. Use of notes, text, and TI-89 **is** authorized.
2. Use pencil.
3. Use **complete sentences.**
4. You **need not** show the details of computations obtained via TI-89. Show all other work.
5. All problems are weighted as indicated; total of 100 points possible.

1. **(5 points.)** Show, **using matrix multiplication**, that $v = (2, -4)$ is an eigenvector for the matrix $\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$. What is the corresponding eigenvalue?

2. Let $T_A : \mathcal{R}^2 \to \mathcal{R}^3$ be defined by $T_A(x_1, x_2) = (x_2, x_1 - 2x_2, -2x_1 + x_2)$

   (a) **(10 points.)** Find the standard matrix representation, A, of T_A. and use it to find $T(3, 4)$.

   (b) **(10 points.)** Find $ker(T_A)$

   (c) **(5 points.)** Determine whether T_A is one-to-one, and state your reasoning.

(d) **(5 points.)** Determine whether T_A is onto, and state your reasoning.

3. Let $A = \begin{pmatrix} -1 & -1 & -1 & 1 & 1 \\ -3 & -3 & 0 & 2 & 1 \\ 1 & 1 & -2 & 0 & 1 \end{pmatrix}$.

(a) **(10 points.)** Find a basis for $null(A)$.

(b) **(10 points.)** Find a basis for $row(A)$.

(c) **(10 points.)** Find a basis for $col(A)$.

4. (a) **(5 points.)** Use your answer to 3(b) to show that $(-3, -3, -6, 4, 5) \in row(A)$.

(b) **(5 points.)** Use your answer to 3(c) to show that $(-2, -5, 1) \in col(A)$.

5. Suppose that $RREF(A) = \begin{pmatrix} 1 & 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

(a) **(5 points.)** What is rank(A)? Explain.

(b) **(5 points.)** What is nullity(A)? Explain.

(c) **(5 points.)** Is $A^T A$ invertible? Why or why not? What about AA^T?

6. **(10 points.)** Show that $v_1 = (1, 0, 1), v_2 = (0, 1, 1)$, and $v_3 = (0, 1, -1)$ form a basis for $\mathcal{R}^3$, or show that they don't. Explain your reasoning.

Extra (10 points.) The matrix for the projection onto $col(A)$, where A is defined in Problem 3, is $Proj_{col(A)} = \frac{1}{46}\begin{pmatrix} 10 & 6 & 18 \\ 6 & 45 & -3 \\ 18 & -3 & 37 \end{pmatrix}$. Find the projection of the vector (1,1,-1) and show that the error vector is orthogonal to $col(A)$.