MA 396 Lesson 12 - Board Problems

Higher-order Taylor Methods

1. Use the Taylor Method of order 2 with $h = .5$ to approximate the solution to

$$y' = 1 - y \qquad 0 \le t \le 2 \qquad y(0) = 0$$

Compute the actual error at $t = 2$. You can use *Mathematica* to find the exact solution. Compare your approximate solution to the actual solution.

$$y'' = -y'(t) = -1 + y$$

$$w_0 = 0$$

$$w_{i+1} = w_i + h\left[(1 - w_i) + \frac{h}{2}(-1 + w_i)\right]$$

$$= w_i + h(1 - w_i) + \frac{h^2}{2}(w_i - 1)$$

see mathematica work sheet

2. Set up the difference equation used to approximate the solution using a Taylor Method of order 2 for

$$y' = -ty + \frac{4t}{y} \qquad 0 \le t \le 1 \qquad y(0) = 1 \qquad h = 0.25$$

$$y'' = -y - ty' + \frac{4y - y'4t}{y^2}$$

$$w_0 = 1$$

$$w_{i+1} = w_i + h\left[-tw_i + \frac{4t}{w_i} + \frac{h}{2}\left(-ty' - w_i + \frac{4w_i - 4ty'}{w_i^2}\right)\right]$$

where $y' = -tw_i + \frac{4t}{w_i}$

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In[1]:= `wi[w_, h_] = w + h (1 - w) + (h^2 / 2) (w - 1)`

Out[1]= $h\,(1-w) + \frac{1}{2}\,h^2\,(-1+w) + w$

In[2]:= `w1 = wi[0, .5]`

Out[2]= 0.375

In[4]:= `w2 = wi[w1, .5]`

Out[4]= 0.609375

In[5]:= `w3 = wi[w2, .5]`

Out[5]= 0.755859

In[6]:= `w4 = wi[w3, .5]`

Out[6]= 0.847412

In[7]:= `sol = DSolve[{y'[t] == 1 - y[t], y[0] == 0}, y, t]`

Out[7]= $\{\{y \to \text{Function}[\{t\}, e^{-t}\,(-1+e^{t})]\}\}$

In[8]:= `y1 = y[.5] /. sol`

Out[8]= {0.393469}

In[10]:= `y2 = N[y[1] /. sol]`

Out[10]= {0.632121}

In[11]:= `y3 = N[y[1.5] /. sol]`

Out[11]= {0.77687}

In[12]:= `y4 = N[y[2] /. sol]`

Out[12]= {0.864665}