

MA396 – Lesson 11
Euler's Method

1. Given $y' = \frac{2}{t}y + t^2e^t$ with $1 \leq t \leq 2$ and $y(1) = 0$, and the exact solution of $y(t) = t^2(e^t - e)$, use Euler's Method with $h = 0.5$ to approximate the solution.

2. Given the same function with $h = 0.1$ and the following table, find $y(2)$.

t	w
1.0	0.0000000
1.1	0.2718282
1.2	0.6847556
1.3	1.2769783
1.4	2.0935477
1.5	3.1874451
1.6	4.6208178
1.7	6.4663964
1.8	8.8091197

Compare to $h = 0.5$.

3. Compute the value of h necessary for $|y(t_i) - w_i| \leq 0.1$ at $t = 2.0$ using Equation 5.10 in the text.

4. Given $y' = 1 + \frac{y}{t}$ with $1 \leq t \leq 2$, $y(1) = 2$, $h = 0.25$, and the exact solution $y(t) = t \ln(t) + 2t$, find the error bound at $t = 2$.

■ Problem 1

In[1]:= $f[y_, t_] = (2/t) * y + t^2 * \text{Exp}[t]$

Out[1]= $e^t t^2 + \frac{2y}{t}$

In[2]:= $w[y_, t_, h_] = y + h * f[y, t]$

Out[2]= $y + h \left(e^t t^2 + \frac{2y}{t} \right)$

In[3]:= $w1 = w[0, 1, .5]$

Out[3]= 1.35914

In[4]:= $w2 = w[w1, 1.5, .5]$

Out[4]= 7.30714

■ Problem 2

In[5]:= $w9 = w[8.8091197, 1.8, .1]$

Out[5]= 11.748

In[6]:= $w10 = w[w9, 1.9, .1]$

Out[6]= 15.3982

■ Problem 3

In[7]:= $d2f[t_] = D[t^2 (\text{Exp}[t] - \text{Exp}[1]), \{t, 2\}]$

Out[7]= $2(-e + e^t) + 4e^t t + e^t t^2$

In[8]:= $M = N[d2f[2]]$

Out[8]= 98.0102

In[9]:= $df = D[(2/t) * y + t^2 * \text{Exp}[t], y]$

Out[9]= $\frac{2}{t}$

In[10]:= $L = 2$

Out[10]= 2

In[11]:= $\text{Err} = ((h * M) / (2 * L)) * (\text{Exp}[L * (2 - 1)] - 1)$

Out[11]= 156.548 h

```
In[12]:= Solve[Err == .1, h]
```

```
Out[12]= {{h -> 0.000638781}}
```

■ Problem 4

```
In[13]:= f[y_, t_] = 1 + y/t
```

```
Out[13]= 1 +  $\frac{y}{t}$ 
```

```
In[14]:= s[t_] = t * Log[t] + 2 t
```

```
Out[14]= 2 t + t Log[t]
```

```
In[15]:= D[s[t], {t, 2}]
```

```
Out[15]=  $\frac{1}{t}$ 
```

```
In[16]:= M = 1
```

```
Out[16]= 1
```

```
In[17]:= D[f[y, t], y]
```

```
Out[17]=  $\frac{1}{t}$ 
```

```
In[18]:= L = 1
```

```
Out[18]= 1
```

```
In[19]:= EB = ((.5 * M) / (2 * L)) * (Exp[L * (2 - 1)] - 1)
```

```
Out[19]= 0.42957
```