# MA 396 Lesson 13 - Board Problems

Runge-Kutta Methods

1. Given the following initial-value problem $y' = 1 + (t - y)^2$ with $2 \le t \le 3$, $y(2) = 1$, and $h = 0.5$, use the Modified Euler method to approximate the solution. Compare to the exact solution of $y(t) = t + \dfrac{1}{1 - t}$

2. Repeat Problem 1 using Heun's method.

3. Repeat Problem 1 using the Midpoint method.

4. Repeat Problem 1 using Runge-Kutta method of order four.

# Lesson 13

## ■ Modified Eulers

```
f[w_, t_] = 1 + (t - w) ^ 2
```

$$1 + (t - w)^2$$

```
y[t_] = t + (1 / (1 - t))
```

$$\frac{1}{1 - t} + t$$

```
M[w_, t_, h_] = w + (h / 2) * (f[t, w] + f[t + h, w + h * f[t, w]])
```

$$w + \frac{1}{2} h \left(2 + (-t + w)^2 + (-h - t + w + h (1 + (-t + w)^2))^2\right)$$

```
w1 = M[1, 2, .5]
```

1.8125

```
Err = Abs[w1 - y[2.5]]
```

0.0208333

## ■ Heun's Method

```
H[w_, t_, h_] = w + (h / 4) * (f[t, w] + 3 * f[t + (2 / 3) * h, w + (2 / 3) * h * f[t, w]])
```

$$w + \frac{1}{4} h \left(1 + (-t + w)^2 + 3 \left(1 + \left(-\frac{2 h}{3} - t + w + \frac{2}{3} h (1 + (-t + w)^2)\right)^2\right)\right)$$

```
w1 = H[1, 2, .5]
```

1.79167

```
Err = Abs[w1 - y[2.5]]
```

0.0416667

## ■ Midpoint Method

```
MM[w_, t_, h_] = w + h * f[t + (h / 2), w + (h / 2) * f[t, w]]
```

$$w + h \left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2} h (1 + (-t + w)^2)\right)^2\right)$$

```
w1 = MM[1, 2, .5]
```

```
1.78125
```

```
Err = Abs[w1 - y[2.5]]
```

```
0.0520833
```

## ■ Runge-Kutta Method

```
k1[w_, t_, h_] = h * f[t, w]
```

$$h\,(1 + (-t + w)^2)$$

```
k2[w_, t_, h_] = h * f[t + (h / 2), w + (k1[w, t, h] / 2)]
```

$$h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\,(1 + (-t + w)^2)\right)^2\right)$$

```
k3[w_, t_, h_] = h * f[t + (h / 2), w + (k2[w, t, h] / 2)]
```

$$h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\,(1 + (-t + w)^2)\right)^2\right)\right)^2\right)$$

```
k4[w_, t_, h_] = h * f[t + h, w + k3[w, t, h]]
```

$$h\left(1 + \left(-h - t + w + h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\,(1 + (-t + w)^2)\right)^2\right)\right)^2\right)\right)^2\right)$$

```
RK[w_, t_, h_] = w + (1 / 6) * (k1[w, t, h] + 2 * k2[w, t, h] + 2 * k3[w, t, h] + k4[w, t, h])
```

$$w + \frac{1}{6}\left(h\,(1 + (-t + w)^2) + 2h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\,(1 + (-t + w)^2)\right)^2\right) + 2h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\,(1 + (-t + w)^2)\right)^2\right)\right)^2\right) + h\left(1 + \left(-h - t + w + h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\left(1 + \left(-\frac{h}{2} - t + w + \frac{1}{2}\,h\,(1 + (-t + w)^2)\right)^2\right)\right)^2\right)\right)^2\right)\right)$$

```
w1 = RK[1, 2, .5]
```

```
1.83332
```

```
Err = Abs[w1 - y[2.5]]
```

$$9.9726 \times 10^{-6}$$