Lesson 14 Board Problems

1. Given the following IVP:

$$y' = 1 + \frac{y}{t}$$

with $1 \le t \le 2$, $y(1) = 2$, use the Runge-Kutta-Fehlberg method to approximate the solution of $y(2)$. Use $hmin = 0.05$, $hmax = 0.25$, and $TOL = 10^{-4}$. Compare to the exact solution of $y(t) = t \ln t + 2t$.

2. Now change the parameters to $hmin = 0.1$, $hmax = 0.2$. What happens?

3. Now change TOL to 10^{-6}. Then to 10^{-8}. What happens?

4. If the desired step size is $hmin = 0.1$, $hmax = 0.25$, what should the tolerance be?

10^{-7} OK, BUT 10^{-8} MINIMUM H IS EXCEEDED.

Problem 1

Runge-Kutta-Fehlberg Method

T(i)	W(i)	H	R
1	2.00000000	0	0
1.25	2.778929884	0.25	5.971833178×10^{-6}
1.5	3.608198493	0.25	2.723877686×10^{-6}
1.75	4.479328784	0.25	1.414126904×10^{-6}
2.	5.386295801	0.25	8.054362375×10^{-7}

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y[t_] = t * Log[t] + 2 * t
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2 t + t Log[t]

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N[y[2]]
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5.386294361

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Error = Abs[5.386295800587068 - y[2]]
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1.439467178×10^{-6}

Problem 2

Runge-Kutta-Fehlberg Method

T(i)	W(i)	H	R
1	2.00000000	0	0

1.2	2.618786105	0.2	2.723877686×10^{-6}
1.4	3.271061557	0.2	1.414126904×10^{-6}
1.6	3.952006388	0.2	8.054362376×10^{-7}
1.8	4.658016714	0.2	4.91834678×10^{-7}
2.	5.386295202	0.2	3.170857516×10^{-7}

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Error = Abs[5.386295201605994 - y[2]]
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8.404861032×10^{-7}

Problem 3

Runge-Kutta-Fehlberg Method

T(i)	W(i)	H	R
1	2.00000000	0	0
1.130771282	2.400514346	0.130771282	5.813871452×10^{-7}
1.256569738	2.800121963	0.1257984561	3.185726016×10^{-7}
1.397223869	3.261801522	0.1406541309	3.259701808×10^{-7}
1.553588062	3.791636135	0.1563641935	3.257048061×10^{-7}
1.727452417	4.399213007	0.1738643549	3.257142436×10^{-7}
1.920774138	5.095292211	0.1933217206	3.257139079×10^{-7}
2.	5.386294668	0.07922586196	7.1151097×10^{-9}

Problem 4

Runge-Kutta-Fehlberg Method

T(i)	W(i)	H	R
1	2.00000000	0	0

Minimal H exceeded