MA 396 Lesson 15 - Board Problems

Multistep Methods

Given: $y' = y - t^2 + 1$ $\quad 0 \le t \le 2$ $\quad y(0)=.5$ $\quad h = 0.2$

Problem 1: Use the **Adams-Bashforth two-step method** to approximate $y(.4)$. This will require you to calculate $y(.2)$ using a one-step method. For this, use the Runge-Kutta method known as the **Midpoint method**.

Problem 2: Use the **Adams-Moulton two-step method**, solving for w_{i+1} explicitly, to approximate $y(.4)$. To make this easier, only solve explicitly for $w(.4)$.

Problem 3: Use the **Adams-Bashforth two-step method** as the predictor and the **Adams-Moulton two-step method** as the corrector to approximate $y(.4)$ using the predictor-corrector technique.

Problem 4: The exact solution is given as $y(t) = (t+1)^2 - 0.5e^t$. Calculate the error bound for the approximations in problems 1 and 2.

■ Midpoint Method

In[2]:=
```
MP[w_, t_, h_] = w + h * f[t + h / 2, w + (h / 2) * f[t, w]]
```

Out[2]= $w + h\, f\left[\frac{h}{2} + t,\ w + \frac{1}{2} h\, f[t, w]\right]$

■ Adams-Bashforth Two Step Method

```
AB2[w1_, t1_, w0_, t0_, h_] = w1 + (h / 2) * (3 * f[t1, w1] - f[t0, w0])
```

Out[3]= $w + \frac{1}{2} h\, (3\, f[t, w] - f[to, wo])$

■ Adams-Moulton Two-Step Implicit Method

In[5]:=
```
AM2[w2_, t2_, w1_, t1_, w0_, t0_, h_] = w1 + (h / 12) * (5 * f[t2, w2] + 8 * f[t1, w1] - f[t0, w0])
```

Out[5]= $w1 + \frac{1}{12} h\, (-f[t0, w0] + 8\, f[t1, w1] + 5\, f[t2, w2])$

■ Problem 1

In[9]:=
```
f[t_, w_] = w - t^2 + 1
```

Out[9]= $1 - t^2 + w$

In[11]:=
```
w1 = MP[.5, 0, .2]
```

Out[11]= 0.828

In[13]:=
```
w2 = AB2[w1, .2, .5, 0, .2]
```

Out[13]= 1.2144

■ Problem 2

In[14]:=
```
WM2 = AM2[w, .4, w1, .2, .5, 0, .2]
```

Out[14]= 0.828 + 0.0166667 (12.804 + 5 (0.84 + w))

In[15]:=
```
Solve[% == w, w]
```

Out[15]= {{w → 1.21244}}

■ Problem 3

Use our solution in Problem 2 as the predictor

```
In[19]:= WM2 = AM2[w2, .4, w1, .2, .5, 0, .2]
Out[19]= 1.2126
```

Problem 4

AB2 Error bound

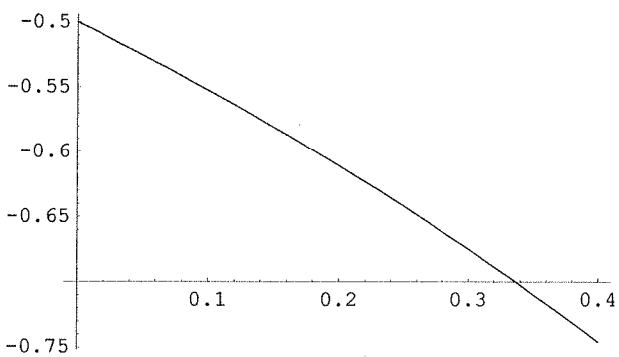
```
In[1]:= y[t_] = (t + 1) ^2 - .5 * Exp[t]
```

Out[1]= $-0.5\, e^{t} + (1+t)^{2}$

```
In[2]:= D3[t_] = D[y[t], {t, 3}]
```

Out[2]= $-0.5\, e^{t}$

```
In[3]:= Plot[D3[t], {t, 0, .4}]
```



Out[3]= - Graphics -

Max error at t = .4

```
In[5]:= EB = Abs[(5 / 12) * D3[.4] * .2 ^2]
Out[5]= 0.0124319
```

AM2 Error Bound

```
In[6]:= D4[t_] = D[y[t], {t, 4}]
```

Out[6]= $-0.5\, e^{t}$

```
In[7]:= EB = Abs[- (1 / 24) * D4[.4] * .2 ^3]
Out[7]= 0.000248637
```